

Calculate the time it takes for a particle to slide from point  $(X_o, Y_o)$  to  $(X_f, Y_f)$  along the frictionless curve  $y=f(x)$ .

Assume the initial velocity is zero and the only force acting on the particle is gravity (and the normal force of the curve).

The loss of potential energy equals the gain in kinetic energy, so the velocity  $V$  along the curve  $y=f(x)$  at any value of  $y$  is given by:

```
(%i1) kill(all)$ y: f(x)$
      (1/2)*M*V^2=M*g*(Y_o-y)$
      eq1: solve(%,V)[2];
```

```
(%o3) V=sqrt(2)*sqrt(g*Y_o-g*f(x))
```

An incremental distance  $ds$  along the curve  $y=f(x)$  is given by  
 $ds = \sqrt{dx^2+dy^2} = dx*\sqrt{1+(dy/dx)^2}$

```
(%i4) ex1: sqrt(1+diff(y,x)^2)$
      ds = dx*ex1;
```

```
(%o5) ds=dx*sqrt((d/d x f(x))^2+1)
```

The time it takes for the particle to travel from  $(X_o, Y_o)$  to  $(X_f, Y_f)$  along the curve  $y=f(x)$  is given by integrating  $ds/V$  from  $x=X_o$  to  $x=X_f$ , so:

```
(%i6) time = integrate(ex1/rhs(eq1),x,X_o,X_f);
```

```
(%o6) time = \frac{\int_{X_o}^{X_f} \sqrt{\left(\frac{d}{dx} f(x)\right)^2 + 1}}{\sqrt{2g(Y_o - f(x))}} dx
```