

SOLUTION:

By symmetry,  $F_{1x}=F_{2x}$  and  $F_{3x}=F_{4x}$ .

By force balance on the vehicle in the X direction,  $F_{1x}+F_{2x} = F_{3x}+F_{4x} \Rightarrow$  all four  $F_x$  forces are equal; call them  $F_x$

By torque balance on each wheel (in the plane of the wheel), each  $F_{ny}$  must equal  $\tau/r$ . Call them  $F_y$ .

By torque balance on the vehicle,  $F_x \cdot \text{wheelbase} = F_y \cdot \text{trackwidth} \Rightarrow F_x = f_2 \cdot F_y$ , where  $f_2 = \text{trackwidth/wheelbase}$ .

The magnitude of the friction force at each wheel is the same, and given by

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{f_2^2 F_y^2 + F_y^2} = F_y \sqrt{1 + f_2^2} = (\tau/r) \sqrt{1 + f_2^2}$$

Since the CoM is aft of the CoG, there is more weight on the rear wheels.

Let  $f_1 = (\text{distance from CoG to CoM}) / (\text{distance from CoG to a point midway between the two rear wheels})$

Then the weight  $N_f$  on each front wheel is given by  $N_f = (W/4) \cdot (1 - f_1)$

The maximum static friction at each front wheel is  $F_{fmax} = \mu \cdot N_f = \mu \cdot (W/4) \cdot (1 - f_1)$

Since the friction force on all four wheels is the same, and the maximum static friction available at the front wheels is less than the rear, the front wheels will slip first. When they do, the vehicle will no longer be in static equilibrium. Given the assumptions we have made about the nature of friction, the vehicle will then begin to rotate. If those assumptions are not exactly true, the vehicle may hop and jerk.

To find the torque at which this occurs, set  $F = F_{fmax}$  and solve for  $\tau$ :

$$F = F_{fmax} \Rightarrow (\tau/r) \sqrt{1 + f_2^2} = \mu \cdot (W/4) \cdot (1 - f_1) \Rightarrow \tau = r \cdot \mu \cdot (W/4) \cdot (1 - f_1) / \sqrt{1 + f_2^2}$$