

List of Variables:

V : Voltage applied at the motor.

I : Current going through the motor.

R : Internal resistance of motor.

ω : Angular velocity of motor or gearbox output, depending on the equation.

K_v : Motor velocity constant; used in connecting angular velocity to voltage.

τ : Torque either at motor or gearbox output, depending on the equation.

K_t : Motor torque constant connecting current and torque.

α : Angular acceleration.

J : Moment of Inertia.

G : Gear ratio.

The equations describing the behavior of an ideal DC motor are as follows:

$$V = IR + \frac{\omega}{K_v} \quad (1)$$

$$\tau = IK_t \quad (2)$$

Given the typical numbers which you are given about a motor (free speed, free current, stall torque, and stall current), it is possible to derive all the necessary constants for the motor physics equations.

For R :

$$\begin{aligned} \omega_{stall} &= 0 \\ V &= I_{stall}R \\ R &= \frac{V}{I_{stall}} \end{aligned}$$

For K_v :

$$\begin{aligned} V &= I_{free}R + \frac{\omega_{free}}{K_v} \\ K_v &= \frac{\omega_{free}}{V - I_{free}R} \end{aligned}$$

For K_t :

$$\begin{aligned} \tau_{stall} &= I_{stall}K_t \\ K_t &= \frac{\tau_{stall}}{I_{stall}} \end{aligned}$$

Given the above physics of DC motors, we can derive the contents of A and B . Because we are operating in the continuous time domain, we do not need to worry about, for instance, that an acceleration over the next time step will create a change in the position that isn't just the velocity multiplied by dt . This allows us to construct the first rows of A and B . We also know that the position itself has no effect on the velocity:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & ? \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ ? \end{bmatrix}$$

For the remaining two values, we must determine how the angular acceleration of the motor depends on the current angular velocity (for A) and the voltage (for B).

We already have an equation for the torque, so using that with I substituted in from (1) and speed and torque are scaled by the gear ratio.

$$\tau = J\alpha$$

$$J\alpha = \frac{IK_t}{G}$$

$$\alpha = \frac{\left(V - \frac{\omega}{K_v G}\right) K_t}{RJG}$$

Where J is the moment of inertia of whatever is being spun. Note that these equations are using the output torque and speed rather than the motor output and speed; therefore a gear ratio G is introduced. This can be broken up into the components which depend on V and those which depend on ω :

$$\alpha = \dot{\omega} = \frac{K_t}{RJG}V - \frac{K_t}{K_v RJG^2}\omega \quad (3)$$

This allows us to fill in A and B :

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{K_t}{K_v RJG^2} \end{bmatrix} \quad (4)$$

$$B = \begin{bmatrix} 0 \\ \frac{K_t}{RJG} \end{bmatrix} \quad (5)$$