A FIRST Encounter with Physics

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March 20, 2010

Introduction

The purpose of this document is to give high school students a basic introduction to physics, using examples and applications from the FIRST Robotics Competition. It is intended to help students who have not yet taken physics to be able to solve some of the physics-related problems that come up in the FRC robot design process. A knowledge of algebra is required; understanding of geometry (particularly sine and cosine) will be helpful.

Perhaps this document will ultimately become a useful resource for learning physics. Who knows, perhaps when every high school in the country has a FRC team this will become the definitive physics text!

We will use the metric system throughout this document, because it makes calculations much easier when we start using complex derived units.

Contents

1 Gear ratios

1.1 Concepts: Force and Torque

Force is a specific term in physics that describes the amout of "push" on an object. When you exert force on an object, it begins to accelerate. This is described by the famous equation

$$\mathbf{F} = m \cdot \mathbf{a}$$

where $\mathbf{F}$ is the force, m is the mass of the object, and $\mathbf{a}$ is the object's accleration. It is important to note that in this equation, $\mathbf{F}$ is the sum of all forces acting on the object. When you stand on the floor, your body exerts a downward force on the floor, and the floor exerts an equal upward force on your body. As a result, $\mathbf{F} = 0$, and you don't move at all. In the metric system, force is measured in Newtons; a Newton is equivalent to $\frac{kg \cdot m}{s^2}$, which is the units of acceleration $\left(\frac{m}{s^2}\right)$ multiplied by mass (kg).

When force is applied to the end of a lever, it produces torque. Mathematically, we say

$$\tau = \mathbf{r} \times \mathbf{F} \tag{1}$$

Here, the symbol $\times$ indicates the cross product of the two vectors, not multiplication. However, if we assume that the force is applied perpendicular to the lever arm, the cross product becomes

$$\tau = r \cdot F \tag{2}$$

which is a simple multiplication. Torque is measured in Newton-meters (abbreviated N-m).

1.2 Example

Suppose we have an arm which requires 100 N-m of torque, but our motor can only provide 1 N-m. Clearly, connecting the motor directly to the arm won't work, so what do we do? The answer is that we use a lever arm, in the form of a gear.

Imagine a gear that is 2 cm (0.02 m) in diameter, as in Figure 1.

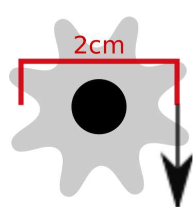

Figure 1: 2 cm gear

The length of the lever arm is the radius of the gear: $\frac{2\,\mathrm{cm}}{2} = 1\,\mathrm{cm}$. If we apply 1 N-m of torque to the center, the force shown by the arrow will be

$$F = \frac{\tau}{r}$$

$$F = \frac{1\,\mathrm{N}\cdot\mathrm{m}}{0.01\,\mathrm{m}} = 100\,\mathrm{N}$$

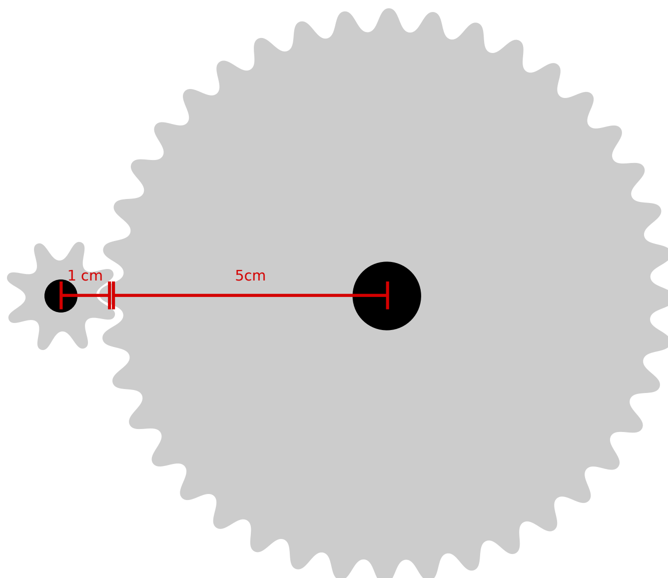

Figure 2: 2 cm gear driving 10 cm gear

In Figure 2, the gear teeth transmit the force to a second gear, 10 cm (0.10 m) in diameter. The torque on this gear will be

$$\tau = 0.05\,\mathrm{m}\cdot 100\,\mathrm{N} = 5\,\mathrm{N}\cdot\mathrm{m}$$

which is a big improvement! To get 100 N-m, we can use additional stages.

As you've probably observed, the change in torque is

$$\tau_{out} = \tau_{in}\cdot\frac{\text{driven gear}}{\text{driving gear}}$$

For example, if an 8-tooth gear drives a 40-tooth gear, the output will have $\frac{40}{8} = 5$ times the torque. Notice, however, that we have to turn the 8-tooth gear around 5 times for the 40-tooth gear to turn around once. So if the motor spins at 1000 RPM, the output will spin at only $1000\cdot\frac{8}{40} = 200$ RPM.

To summarize: when a little gear drives a big gear, the result is slower but has more torque. When a big gear drives a little gear, the result is faster but has less torque.

2 Picking a Motor

2.1 Concepts: Motor characteristics, Part 1

There are four important characteristics we need to know for every motor:

- Stall torque, the amount of torque the motor produces when stalled. This will always be the maximum torque the motor can produce.
- Stall current, the amount of electrical current the motor draws when stalled. This will always be the maximum current.
- Free speed, the speed at which the motor spins when nothing is connected to it. This will be the motor's maximum speed.
- Free current, the amount of current the motor draws when spinning freely. This will be the minimum current.

These values are typically given on the motor's datasheet, and are valid only for a particular voltage (most often 12 V). The 2.5" CIM motor, for example, has the values shown in Table 1.

Stall Torque	Stall Current	Free Speed	Free Current
N-m	Amps	RPM	Amps
2.43	133	5,310	2.7

Table 1: CIM Motor characteristics (Taken from Appendix A)

The performance between these maximum and minimum values is linear, which makes it easy to work with them. A plot of speed and current as a function of torque is shown in Figure 3.

2.2 Examples

2.2.1 Kicker winch

Suppose we have a winch designed to pull back a kicking mechanism, which we need to drive with one of the kit motors. For now, let's select the Fisher-Price motor with the plastic gearbox. The cable wraps around a winch barrel which is 4 cm in diameter.

Looking at the table in Appendix A on page 8, the Fisher-Price motor by itself has a stall torque of 0.45 N-m. The gearbox has a 139:1 gear ratio, meaning that the motor turns around 139 times for each time the white output shaft turns around once. The output shaft torque is simply the input torque multiplied by the gear ratio:

$$\tau_{out} = 0.45\,\mathrm{N}\cdot\mathrm{m}\cdot 139 = 63\,\mathrm{N}\cdot\mathrm{m}$$

We can find the force by dividing by the length of the lever arm:

$$F = \frac{63\,\mathrm{N}\cdot\mathrm{m}}{0.020\,\mathrm{m}} = 3150\,\mathrm{N}$$

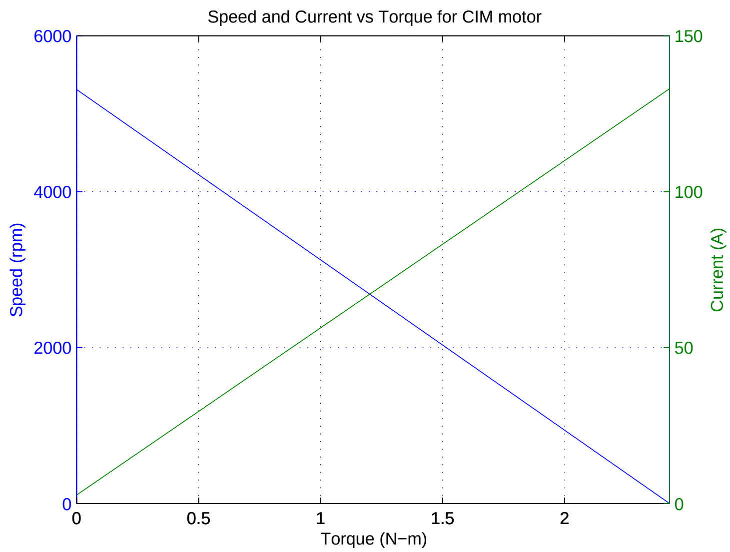

Figure 3: Speed and torque as a function of current for the CIM motor

That's equivalent to approximately 700 pounds, a LOT of force! Now, there's no way we'd actually get this much torque out of the motor, for several reasons:

- We lose power to friction in the gearbox. For a single well-designed gearing stage, the loss is between 5% and 10%. The Fisher-Price gearbox uses cheap plastic gears and has 4 stages, so we'll estimate its overall efficiency at 60%.
- We don't want to run the motor anywhere near stall torque: when the motor stalls, all of the electrical energy going into it is converted to heat. Within a few seconds, the motor will begin to smoke.
- At stall, the Fisher-Price motor will draw 70 Amps, which will trip the breaker. We need to use less than 40 A, and would prefer to use even less than that. As mentioned earlier, torque and current are linearly related, so we can find the torque at a particular current using Equation 3.

$$\text{Torque} = \frac{\text{Desired current}}{\text{Stall current}} \cdot \text{Stall torque} \tag{3}$$

Let's try using these values:

$$\tau = \frac{10\,\text{A}}{70\,\text{A}} \cdot 0.45\,\text{N}\cdot\text{m} = 0.064\,\text{N}\cdot\text{m}$$

$$\tau_{out} = 0.064\,\text{N}\cdot\text{m} \cdot 139 \cdot \frac{60\%}{100\%} = 5.4\,\text{N}\cdot\text{m}$$

$$F = \frac{5.4\,\mathrm{N}\cdot\mathrm{m}}{0.020\,\mathrm{m}} = 270\,\mathrm{N}$$

This is equivalent to 61 pounds, which should be sufficient to drive our winch.

A Motor comparison table

Motor	Technical name	Max Power	Stall Torque	Stall Current	Free Speed	Free Current	Max Efficiency
		Watts	N-m	Amps	RPM	Amps	
CIM	FR-801-001	337	2.43	133	5,310	2.7	65%
Fisher-Price	9015	184	0.45	70	15,600	1.2	68%
Nippon-Denso	262100-3030	23	10.6	18.6	84	1.8	24%