A FIRST Encounter with Physics

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Introduction

The purpose of this document is to give high school students a basic introduction to physics, using examples and applications from the FIRST Robotics Competition. It is intended to help students who have not yet taken physics to be able to solve some of the physics-related problems that come up in the FRC robot design process. A knowledge of algebra is required; understanding of geometry (particularly sine and cosine) will be helpful.

Perhaps this document will ultimately become a useful resource for learning physics. Who knows, perhaps when every high school in the country has a FRC team this will become the definitive physics text!

We will use the metric system throughout this document, because it makes calculations much easier when we start using complex derived units.

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1 Gear ratios

1.1 Concepts: Force and Torque

Force is a specific term in physics that describes the amout of "push" on an object. When a force is exerted on an object - that is, when an object is pushed - it begins to accelerate. This is described by the famous equation

$$\mathbf{F} = m \cdot \mathbf{a} \tag{1}$$

where $\mathbf{F}$ is the force, m is the mass of the object, and $\mathbf{a}$ is the object's accleration. It is important to note that in this equation, $\mathbf{F}$ is the sum of all forces acting on the object. When you stand on the floor, gravity exerts a downward force on your body, and the floor exerts an equal upward force on your body. As a result, the sum of the forces on your body equals 0, and your body doesn't move. In the metric system, force is measured in Newtons; a Newton is equivalent to a $\frac{kg \cdot m}{s^2}$, which is the units of acceleration $\left(\frac{m}{s^2}\right)$ multiplied by mass (kg).

When force is applied to the end of a lever, it produces torque. Mathematically, we say

$$\tau = \mathbf{r} \times \mathbf{F} \tag{2}$$

Here, the symbol $\times$ indicates the cross product of the two vectors, not multiplication. However, if we assume that the force is applied perpendicular to the lever arm, the cross product becomes

$$\tau = r \cdot F \tag{3}$$

which is a simple multiplication. Torque is measured in Newton-meters (abbreviated N-m).

1.2 Example

1.2.1 Arm Torque

Suppose we have a robot arm as shown in Figure 1. The arm is 1 meter long from the pivot to the center of the ball, and the ball has a mass of 5 kilograms. The acceleration due to gravity is 9.8 $\frac{m}{s^2}$. For the moment, we will make the (obviously false) assumption that the arm doesn't weigh anything, because it simplifies the problem. How much torque do we need to lift the arm?

The worst case will occur when the arm is horizontal and trying to lift the ball straight up, as shown in Figure 1. First, we find the downward force on the ball due to gravity, using Equation 1:

$$F = 5\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 49\,\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = 49\,\mathrm{N}$$

For the ball to not move, the forces on the ball must cancel out, which means that the arm must exert a force of 49 Newtons upward. We can use this information in Equation 3 to find the torque at the arm joint:

$$\tau = 1\,\mathrm{m} \cdot 49\,\mathrm{N} = 49\,\mathrm{N} \cdot \mathrm{m}$$

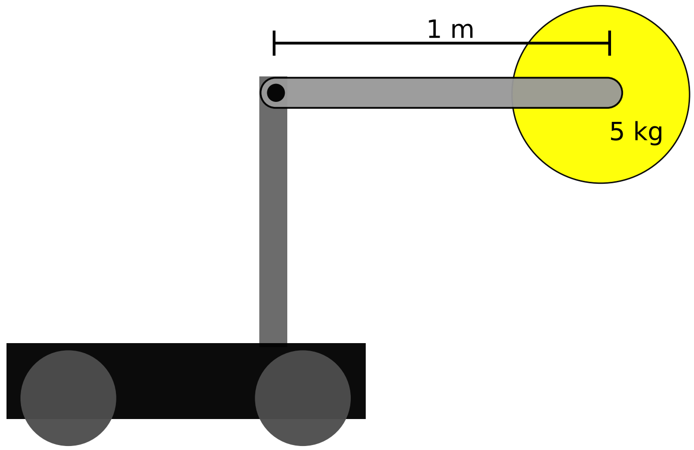

Figure 1: Example robot arm

Of course, our goal is not to hold the ball still - we want to lift it! Thus, Since our goal is actually to lift the ball, we need to provide more than 49 Newton-meters of torque.

1.2.2 Arm Torque, Part 2

In the previous section, we made the assumption that the arm was massless - it didn't weigh anything, and so it didn't require any torque to lift. Obviously, every arm weighs something, so it's important to include the arm in our torque calculations.

If we assume that the arm has a constant linear weight - that is, it doesn't get thinner or lighter towards one end - then we can take the total mass of the arm and act as if that mass is located at a point halfway along the length of the arm. Suppose that in Figure 1 the arm weighs 8 kg. Thus, the downward force on the arm due to gravity is:

$$F = 8\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 78\,\mathrm{N}$$

The halfway point on the arm is $\frac{1\,\mathrm{m}}{2} = 0.5\,\mathrm{m}$, which means that the torque will be

$$\tau = 0.5\,\mathrm{m} \cdot 78\,\mathrm{N} = 39\,\mathrm{N} \cdot \mathrm{m}$$

We can calculate the total torque at the arm joint simply by adding the torque from the ball and the torque from the arm:

$$\tau_{total} = 49\,\mathrm{N} \cdot \mathrm{m} + 39\,\mathrm{N} \cdot \mathrm{m} = 88\,\mathrm{N} \cdot \mathrm{m}$$

If the arm has several sections of different size and weight, we can treat them each as separate pieces and calculate the torque for each one, just like we treated the ball and arm as separate pieces and calculated the total torque by adding them together.

1.2.3 Gearing

Suppose now that we have to power the arm with a motor which can only provide 1 N-m of torque. Clearly, connecting the motor directly to the arm won't work, so what do we do? The answer is that we use a lever arm, in the form of a gear.

Imagine a gear that is 2 cm (0.02 m) in diameter, as in Figure 2.

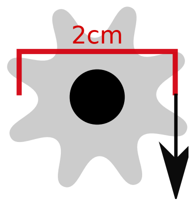

Figure 2: 2 cm gear

The length of the lever arm is the radius of the gear: $\frac{2\,\mathrm{cm}}{2} = 1\,\mathrm{cm}$. If we apply 1 N-m of torque to the center, the force shown by the arrow can be found by rearranging Equation 3:

$$F = \frac{\tau}{r}$$

$$F = \frac{1\,\mathrm{N}\cdot\mathrm{m}}{0.01\,\mathrm{m}} = 100\,\mathrm{N}$$

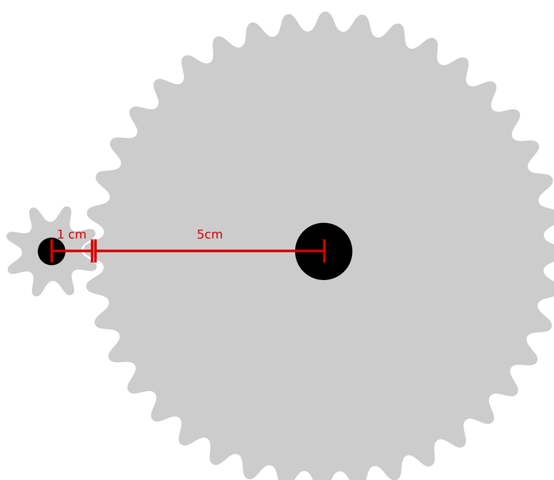

Figure 3: 2 cm gear driving 10 cm gear

In Figure 3, the gear teeth transmit the force to a second gear, 10 cm (0.10 m) in diameter. The torque on this gear will be

$$\tau = 0.05\,\mathrm{m}\cdot 100\,\mathrm{N} = 5\,\mathrm{N}\cdot\mathrm{m}$$

which is a big improvement! To get a torque greater than 49 N-m, we can use additional stages, as shown in Figure 4.

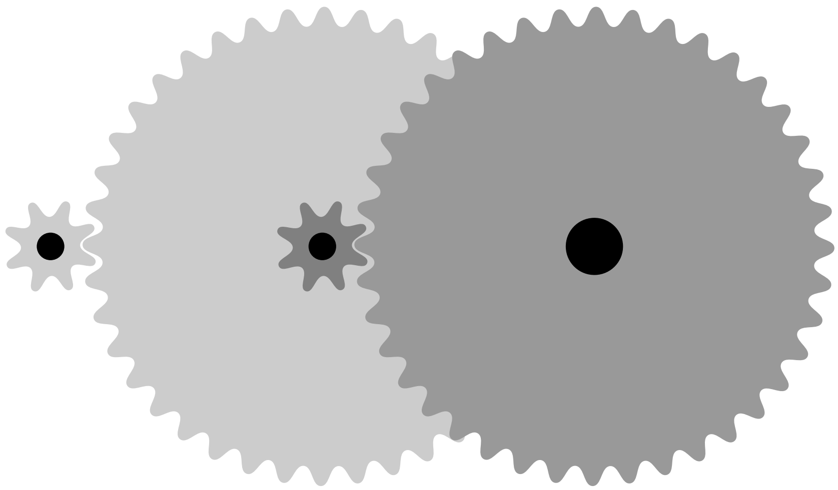

Figure 4: Two-stage gearing. The first small gear drives a large gear, which is on the same shaft as a second small gear. The second gear drives another large gear, producing an overall gear ratio of 25:1.

As you've probably observed, the change in torque is

$$\tau_{out} = \tau_{in} \cdot \frac{\text{driven gear}}{\text{driving gear}} \tag{4}$$

For example, if an 8-tooth gear drives a 40-tooth gear, the output will have $\frac{40}{8} = 5$ times the torque. Notice, however, that we have to turn the 8-tooth gear around 5 times for the 40-tooth gear to turn around once. So if the motor spins at 1000 RPM, the output will spin at only $1000 \cdot \frac{8}{40} = 200$ RPM. This is referred to as a 5:1 gear ratio, since five turns of the input produces one turn of the output.

To summarize: when a little gear drives a big gear, the result is slower but has more torque. This is referred to as "gearing down", because it results in a lower speed. When a big gear drives a little gear, the result is faster but has less torque. This is referred to as "gearing up". The same concept applies when using sprockets and chain or pulleys and belts.

2 Picking a Motor

Picking the right motors for each mechanism is an important step in building a high-performance robot. By doing a little planning, you can squeeze the best performance out of each motor.

2.1 Concepts: Motor characteristics, Part 1

There are four important characteristics we need to know for every motor:

- Stall torque, the amount of torque the motor produces when stalled. This will always be the maximum torque the motor can produce.
- Stall current, the amount of electrical current the motor draws when stalled. This will always be the maximum current.
- Free speed, the speed at which the motor spins when nothing is connected to it. This will be the motor's maximum speed.
- Free current, the amount of current the motor draws when spinning freely. This will be the minimum current.

These values are typically given on the motor's datasheet, and are valid only for a particular voltage (most often 12 V). The Fisher-Price motor, for example, has the values shown in Table 1.

Stall Torque	Stall Current	Free Speed	Free Current
N-m	Amps	RPM	Amps
0.45	70	15,600	1.2

Table 1: Fisher-Price Motor characteristics (Taken from Appendix A)

The performance between these maximum and minimum values is linear, which makes them easy to work with. A plot of speed and current as a function of torque is shown in Figure 5.

2.2 Example: Kicker winch

Suppose we have a winch designed to pull back a kicking mechanism, which we need to drive with one of the kit motors. For now, let's select the Fisher-Price motor with the plastic gearbox. The cable wraps around a winch barrel which is 4 cm in diameter.

Looking at the table in Appendix A on page 12, the Fisher-Price motor by itself has a stall torque of 0.45 N-m. The gearbox has a 139:1 gear ratio, meaning that the motor turns around 139 times for each time the white output shaft turns around once. The output shaft torque is simply the input torque multiplied by the gear ratio:

$$\tau_{out} = 0.45\,\mathrm{N} \cdot \mathrm{m} \cdot 139 = 63\,\mathrm{N} \cdot \mathrm{m}$$

We can find the force on the cable by dividing by the length of the lever arm (the radius of the winch barrel):

$$F = \frac{63\,\mathrm{N} \cdot \mathrm{m}}{0.020\,\mathrm{m}} = 3150\,\mathrm{N}$$

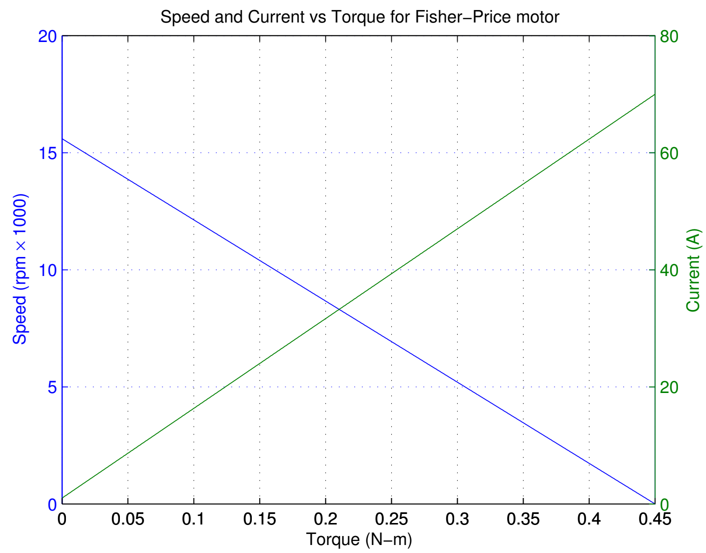

Figure 5: Speed and current as a function of torque for the Fisher-Price motor

That's equivalent to approximately 700 pounds, a LOT of force! Now, there's no way we'd actually get this much torque out of the motor, for several reasons:

- We lose power to friction in the gearbox. For a single well-designed gearing stage, the loss is between 5% and 10%. The Fisher-Price gearbox uses cheap plastic gears and has 4 stages, so we'll estimate its overall efficiency at 60%.
- We don't want to run the motor anywhere near stall torque: when the motor stalls, all of the electrical energy going into it is converted to heat. Within a few seconds, the motor will begin to smoke.
- At stall, the Fisher-Price motor will draw 70 Amps, which will trip the breaker. We need to use less than 40 A, and would prefer to use even less than that. As mentioned earlier, torque and current are linearly related, so we can find the torque at a particular current using Equation 5.

$$\text{Torque} = \frac{\text{Desired current}}{\text{Stall current}} \cdot \text{Stall torque} \tag{5}$$

Let's redo the calculations, using a target current of 10 Amps and a gearbox efficiency of 60%:

$$\tau = \frac{10\,\mathrm{A}}{70\,\mathrm{A}} \cdot 0.45\,\mathrm{N}\cdot\mathrm{m} = 0.064\,\mathrm{N}\cdot\mathrm{m}$$

$$\tau_{out} = 0.064\,\mathrm{N}\cdot\mathrm{m} \cdot 139 \cdot \frac{60\%}{100\%} = 5.4\,\mathrm{N}\cdot\mathrm{m}$$

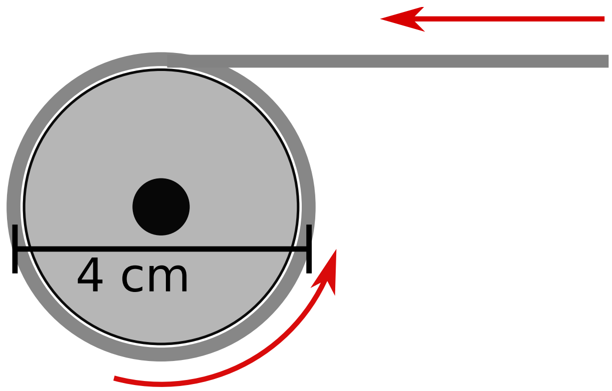

Figure 6: Diagram of kicker winch example

$$F = \frac{5.4\,\mathrm{N} \cdot \mathrm{m}}{0.020\,\mathrm{m}} = 270\,\mathrm{N}$$

This is equivalent to 61 pounds, which should be sufficient to drive our winch.

2.3 Concepts: Work and Power

In physics, energy is commonly defined as the ability to do work. You're probably aware that there are many forms of energy: chemical energy, heat, light, motion, and so forth. In the metric system, one unit for energy is the Joule, which is the energy required to exert 1 Newton of force for a distance of 1 meter.

Suppose we want to lift our robot 1.5 meters off the ground. How much energy does that require?

The downward force the robot exerts due to gravity is given by

$$F = m \cdot g$$

where g is the gravitational constant on Earth, roughly $9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$. The value m is the mass of the robot. Assuming a 150 pound (68 kg) robot,

$$F = 68\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 670\,\mathrm{N}$$

and energy is given by force multiplied by the distance:

$$energy = F \cdot x$$

which gives us

$$energy = 670\,\mathrm{N} \cdot 1.5\,\mathrm{m} = 1000\,\mathrm{J}$$

So it takes 1000 Joules to lift our robot. We traditionally combine these two equations and write

$$PE = m \cdot g \cdot h \tag{6}$$

where the term PE stands for "potential energy".

In everyday usage, we treat the terms "power" and "strength" as synonomous. In physics, however, power has a specific definition: the amount of work done in a specific amount of time. In other words, it's the number of Joules used in one second. This unit is named the Watt (abbreviated W).

Suppose we want our lifting mechanism to do its job in 2 seconds. How much power is required?

We can use equation 7, where P is the power required, w is the work done, and t is the time it takes to do the work.

$$P = \frac{w}{\Delta t} \tag{7}$$

Plugging in the values we have, we get:

$$P = \frac{1000\,\mathrm{J}}{2.0\,\mathrm{s}} = 500\,\mathrm{W}$$

The concept of power is particularly important when choosing a motor: we can gear a motor down to get more torque, or gear it up to get more speed, but we can't do anything (that would be FIRST-legal) to increase the maximum power output of the motor.

2.4 Concepts: Motor characteristics, Part 2

2.4.1 Output power, input power, and efficiency

How do we determine the motor's output power? It goes back the same physics of force and work.

As stated before, energy is given by force multiplied by distance, which we state formally with Equation 8. The variable w is work (the amount of energy used), $\mathbf{F}$ is the force applied, and x is the distance traveled.

$$w = \mathbf{F} \bullet x \tag{8}$$

Imagine a lever arm attached to the motor. We prefer easy numbers, so we'll define it to be 1 m long. The motor is spinning at some speed in RPM, which we'll call r. Every time the motor spins around, the lever arm travels

$$2 \cdot \pi \cdot r = 2 \cdot \pi\,\mathrm{m}$$

We can easily convert from RPM to revolutions per second:

$$RPS = \frac{RPM}{60} = \frac{r}{60}$$

which means that the tip of the lever arm travels

$$\frac{r}{60} \cdot 2 \cdot \pi\,\mathrm{m} \tag{9}$$

each second. Now we need to calculate the force at the end of the lever arm, which we can do with Equation 3.

$$F = \frac{\tau}{1\,\mathrm{m}} \tag{10}$$

Now we use Equation 8 to combine Equations 9 and 10:

$$w = \frac{\tau}{1\,\mathrm{m}} \cdot \frac{r}{60} \cdot 2 \cdot \pi\,\mathrm{m}$$

And because the distance we calculated is the distance the arm moves in 1 second, we divide by 1 second to get the output power, which gives us Watts.

$$P = \tau \cdot \frac{r}{60} \cdot 2 \cdot \pi\,\mathrm{W}$$

Note that the torque, τ, and the speed, r, are related to each other. To find the power, we have to pick a value for one of them, determine the other, and then put them in the equation.

The input power is electrical, and thus it is given by Equation 11, where P is the power, I is the current, and V is the voltage.

$$P = I \cdot V \tag{11}$$

The efficiency of the motor is simply the percentage of the electrical input power that gets turned into mechanical output power. Mathematically,

$$\eta = \frac{P_{out}}{P_{in}}$$

Generally, we don't manually calculate input power, output power, and efficiency when we're working with a motor. Like good engineers, we make a computer do the hard work once, and then look it up in a table or graph.

2.4.2 Motor curves

The motor data discussed above is almost always plotted on a graph, known as the motor curve. Figure 5 is one example of a motor curve, which you may find on some datasheets.

A Motor comparison table

Motor	Technical name	Max Power	Stall Torque	Stall Current	Free Speed	Free Current	Max Efficiency
		Watts	N-m	Amps	RPM	Amps	
CIM	FR-801-001	337	2.43	133	5,310	2.7	65%
Fisher-Price	9015	184	0.45	70	15,600	1.2	68%
Nippon-	262100-3030	23	10.6	18.6	84	1.8	24%

Glossary

Electrical current is the flow of electrons through a wire, measured in Amps.

Force describes the amount of push on an object, and is measured in Newtons. Force causes a change in motion, described by the equation $\mathbf{F} = m \cdot \mathbf{a}$.

Gearing down is using a small gear, sprocket, or pulley to drive a larger gear, sprocket, or pulley, resulting in lower speed but higher torque.

Gearing up is using a large gear, sprocket, or pulley to drive a smaller gear, sprocket, or pulley, resulting in higher speed but lower torque.

RPM Revolutions Per Minute, a measurement of rotational speed. 1 revolution is one 360-degree turn of a shaft.

Stall occurs when a motor can't produce enough torque to turn and stops moving. This is generally undesireable, and can cause some motors to burn out.

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