2015 FRC District Qualification Points Formula Breakdown

BACKGROUND

The FRC 2015 District Qualification Points Formula was developed out of a need to replace the win-loss-tie (WLT) point value system for District Qualification Ranking use when no WLT system is in place, such as in the 2015 FRC Game, "Recycle Rush." To be clear, this formula is only intended to be used to replace the traditional point system used during qualification matches (i.e. non-playoff matches) – points earned for advancement during the playoff matches and for awards are not affected by this formula.

Upon analyzing 2014 FRC District data, it was discovered that the overall District Qualification results closely approximated a normally distributed set (also known as a Gaussian "bell curve"). Figure 1 shows the combined point awards during all Qualification matches in all District competitions (except for Championship events). The data is "binned" to remove "noise" from infrequent "odd-valued" scores due to ties; scores resulting in 1 and 2 points are combined to 2, scores 3 and 4 are combined to 4, and so on. This similarity in the data to a normal distribution is likely due to the natural result of a WLT system; in order for one team to win another team has to lose, and the points were specifically awarded based on the number of wins a team earned within a bounded number of matches played by all teams. In 2014, the method of awarding district qualification points involved awarding a team with 2 points for each WIN, 1 point for each TIE, and 0 points for a loss. A few of the teams will win most of their matches, a few of the teams will lose most of their matches, but a high number of teams will win/lose about half of their matches.

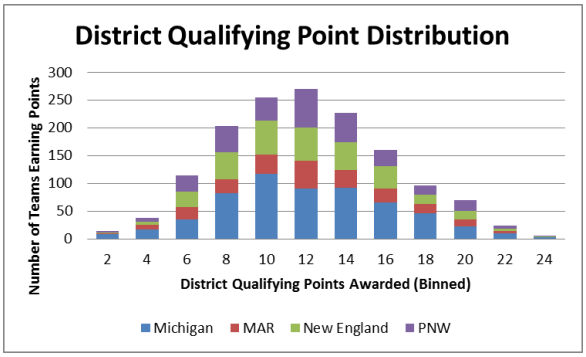

Figure 1: Combined 2014 District Qualifying Point Distribution

SOLUTION

Just as in the WLT based system, the points awarded by a new District Qualification system needed to be based on team performance. Rank has served as the "ultimate" measure of performance of a team over the course of qualification matches, and is common to WLT and non-WLT systems, so Rank was decidedly used as the basis for awarding qualification points. Approximating the WLT system with Rank has its challenges, however, such as the problem of how to award a normally distributed points based on a linear scale (remember, rank is linear from 1 to N where N is the number of teams at a tournament). In order to provide this translation, the Inverse Error Function is used.

The Inverse Error Function (InvERF) is a special function used in probability and statistics. Mathematically the InvERF can be a complicated function to understand, but in practical use the InvERF can be used as a powerful tool for translating a linear set of data into a normally distributed set. For example, in Monte Carlo simulations a set of normally distributed random numbers is often needed; in

order to generate this normally distributed set the Inverse Error Function is often used to translate the linear set of random numbers into a normally distributed set. Similarly, for calculating the District Qualification Points, the InvERF can be used to map the linear Rank into a normally distributed set of point values.

The formula chosen to calculate the District Qualification Points is:

$$DistrictQualificationPoints(N, R, \alpha) = \left\lceil InvERF\left(\frac{N - 2R + 2}{\alpha N}\right)\left(\frac{10}{InvERF\left(\frac{1}{\alpha}\right)}\right) + 12 \right\rceil$$

Where N is the number of Teams at an event (event size), R is a Team's final Rank at the end of Qualification, and α is a term used to control the "flatness" of the distribution of the resulting data set. Factors N and R will be set based upon the tournament size and an individual team's performance, but α has been empirically set to 1.07 by the District Qualifying Points committee based on team point distributions for varying sizes of events.

In the formula, the ⌈ ⌉ symbols represent the "ceiling" function, which mean to "round up to the next whole integer." This provides integer solutions through the valid range of point values.

The first element in the equation, the first multiplicand $InvERF\left(\frac{N-2R+2}{\alpha N}\right)$, provides a scaling factor to represent the Team Rank over the possible number of Teams (N) while also scaling the value across the range [-1, 1] for the Inverse Error Function. A simpler form of $InvERF\left(\frac{2R-N}{\alpha N}\right)$ does this as well, but assigns the highest value to the Nth ranked team instead of the 1st ranked team, thus a substitution of R=(N-R+1) resulting in the provided formula provides the proper reversal.

The second element in the equation, the second multiplicand $\left(\frac{10}{InvERF\left(\frac{1}{\alpha}\right)}\right)$, is a scaling factor for the normally distributed values. This multiplicand scales the data proportionally so that the maximum value is always 10. The factor 10, which sets the positive and negative range of the data set, was chosen to minimize "gaps" between values awarded.

The third element in the equation, + 12, shifts the data so that the maximum point value (for the first ranked team) is 22. The resulting points provides a similar total tournament award (the total number of points that can be given out at a given event of a given size) as in the 2014 FRC season.

CONCLUSION

The goal of this effort was to find a means of approximating the 2014 WLT qualification points award method using a tangible performance metric for 2015. The methodology of using Rank (based on the number of teams attending a tournament) to award a normally distributed amount of points does this.

In Figure 2 we see a comparison between the actual point values awarded during Qualification matches in District events in the 2014 season and what would have been awarded if the District Qualification Points formula were used (marked as the "Distribution"), per District and combined. This provides some indication that while teams may score slightly higher points under the calculated methodology, the resultant distributions and awarded points are comparable.

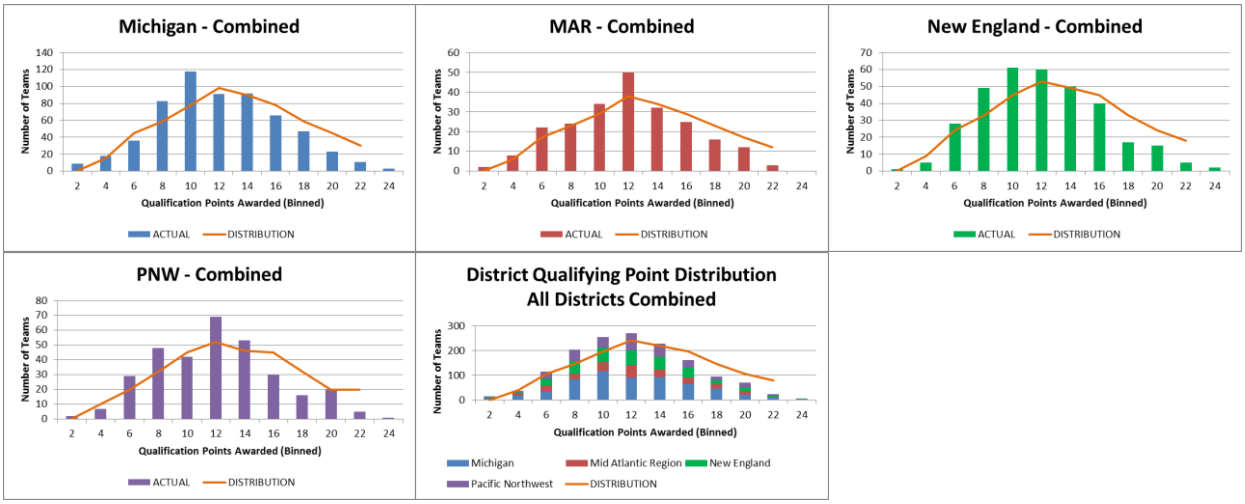

Figure 2 – District Actuals vs Calculated Point Distributions

The representative point tables shown below demonstrate the point awards for Ranks for a few different event sizes.

Table 1 – 32-Team Tournament Points Award by Rank

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Points	22	21	20	19	18	17	17	16	16	15	15	15	14	14	13	13	12	12	12	11	11	10	10	10
Rank	25	26	27	28	29	30	31	32																
Points	9	9	8	8	7	6	5	4																

Table 2 - 40-Team Tournament Points Award by Rank

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Points	22	21	20	19	19	18	18	17	17	16	16	16	15	15	14	14	14	13	13	13	12	12	12	12
Rank	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40								
Points	11	11	11	10	10	9	9	9	8	8	7	7	6	6	5	4								

Table 3 - 55-Team Tournament Points Award by Rank

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Points	22	21	21	20	19	19	18	18	18	17	17	17	16	16	16	16	15	15	15	15	14	14	14	14
Rank	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Points	13	13	13	13	12	12	12	12	11	11	11	11	10	10	10	10	9	9	9	9	8	8	8	7
Rank	49	50	51	52	53	54	55																	
Points	7	7	6	6	5	4	4																	

Table 4 - 60-Team Tournament Points Award by Rank

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Points	22	22	21	20	20	19	19	18	18	18	17	17	17	16	16	16	16	15	15	15	15	14	14	14
Rank	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Points	14	14	13	13	13	13	12	12	12	12	12	11	11	11	11	11	10	10	10	10	9	9	9	9
Rank	49	50	51	52	53	54	55	56	57	58	59	60												
Points	8	8	8	7	7	7	6	6	5	5	4	3												

Table 5 - 64-Team Tournament Points Award by Rank

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Points	22	22	21	20	20	19	19	18	18	18	17	17	17	17	16	16	16	16	15	15	15	15	15	14
Rank	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Points	14	14	14	13	13	13	13	13	12	12	12	12	12	12	11	11	11	11	10	10	10	10	10	9
Rank	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64								
Points	9	9	9	8	8	8	8	7	7	7	6	6	5	5	4	3								