

Math Quiz 8
No trig solution

See **Figure1** for definitions of A, B, C, D, and E.

AD is the bisector of angle CAB.

Triangles DEB and ACB are similar.

The problem gives ratio K and asks you to solve for ratio AB/CB. So without loss of generality, let CD=1.

See **Figure2**.

The red triangle and triangle ADC are similar.

$$\text{Therefore } AD/CD = (R+r)/(R-r) = (Kr+r)/(KR-r) = (K+1)/(K-1)$$

$$\text{so } AD = (K+1)/(K-1) * CD$$

$$AC = \sqrt{AD^2 - CD^2}$$

Triangles AED and ACD are congruent, so

$$AE = AC$$

$$DE = CD$$

$$(CD+DB)/AC = \sqrt{DB^2 - DE^2}/DE \quad // \text{ similar triangles DEB and ACB}$$

Solve above equation for DB

$$CB = CD+DB$$

$$EB = \sqrt{DB^2 - DE^2}$$

$$AB = AE+EB$$

$$\text{answer} = AB/CB$$

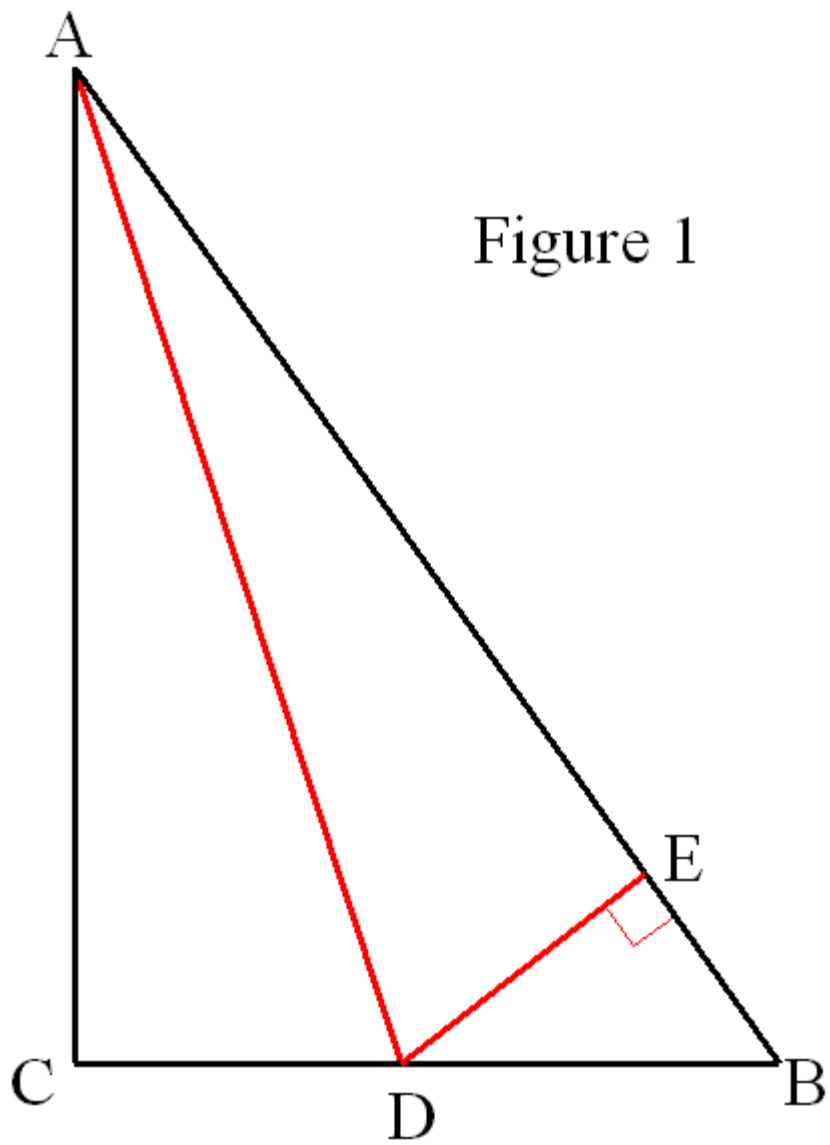


Figure 1

Figure 2

