Derivation of an algorithm for computing the profile for a "bump-less" mecanum roller

Mecanum wheel with 45° rollers

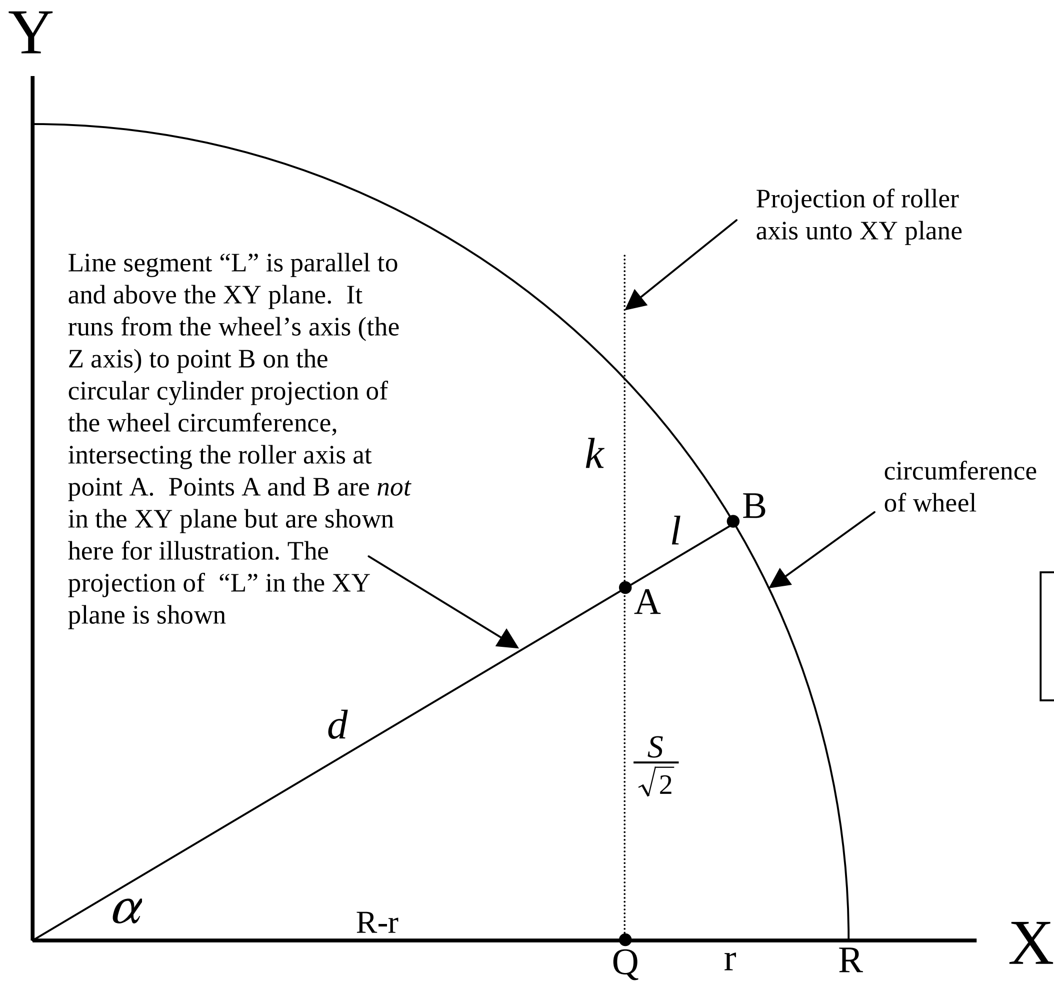
Wheel is lying in the XY plane

R wheel radius

r roller max radius

S distance along roller axis to intersection with Line "L"

θ angle between Line "L" and the roller axis (this angle is not in the XY plane)



Pick a value for **R** and a value for **r**,

Then make the following computations as you vary the value of the parameter S from zero to half the length of the roller axis:

$$\alpha = \tan^{-1}\left(\frac{S/\sqrt{2}}{(R-r)}\right)$$

$$d = \frac{R-r}{\cos(\alpha)}$$

$$l = R - d$$

$$\theta = \cos^{-1}\left(\frac{\sin(\alpha)}{\sqrt{2}}\right)$$

See notes on following page

$$\boxed{h = S + \frac{l\sin(\alpha)}{\sqrt{2}} \qquad Rr = l\sin(\theta)}$$

The above are parametric equations. S is the parameter. Pick values for S and compute a table of *Rr* vs *h*. "*Rr*" is the roller radius at a distance "*h*" from the center of the roller measured along the roller axis

See diagram on previous page.

Let $\hat{\mathbf{u}}_1$ be a unit vector lying on line L with origin at point A and x,y,z components cos(α), sin(α), zero, respectively.

Let $\hat{\mathbf{u}}_2$ be a unit vector lying on the roller axis with origin at point A and x,y,z components zero, cos(45), sin(45), respectively.

Let θ be the angle between these two unit vectors.

Then the dot product of the unit vectors must be cos(θ) = sin(α)*cos(45), so

θ = acos(sin(α)/sqrt(2))

Drop a perpendicular from point B to the roller axis at point C (not shown). The length of BC is *l**sin(θ) and is the radius ***Rr*** of the roller at point C on its axis. Point C is a distance ***h*** from the midpoint Q of the roller axis. The length of QC equals QA+AC. The length of QA is S. The length of AC is *l**cos(θ), which equals *l**sin(θ)/sqrt(2). So ***h*** = QC = S + *l**sin(θ)/sqrt(2).

With a little bit of algebra, the parametric equations derived on the previous pages can be reduced to an algorithm requiring no trigonometry. Here's the algorithm:

Chose a value for R (the radius of the wheel), and r (the radius of the roller at the midpoint of its axis – the "fattest" part of the roller).

Let D=R-r.

Decide what you want the length of the roller axis to be.

Vary "S" from zero to half the length of the roller axis, and repeat the following calculations (in the order shown) for each S to make a table of ***Rr*** vs ***h***:

$$F = \sqrt{2 \cdot D^2 + S^2} \qquad G = \sqrt{4 \cdot D^2 + S^2} \qquad T = \frac{\sqrt{2} \cdot R}{F}$$

$$\boxed{h = \frac{S}{2} \cdot (T + 1) \qquad R_r = \frac{G}{2} \cdot (T - 1)}$$

R_r is the radius of the roller at a distance "h" from the center of the roller

If your CAD program cannot easily import a user-defined profile as a set of XY data points, the following algorithm will create a simple formula for the profile. It will not be as accurate, but it will be very close:

Let “R” be the radius of the mecanum wheel

Let “r” be the radius of the roller at the midpoint (the “fattest” portion of the roller)

Let “L” be the length of the roller

Perform the following calculations in the order shown:

D = R – r

F = sqrt(2*D^2+(L/4)^2)

G = sqrt(4*D^2+(L/4)^2)

T = R*sqrt(2)/F

a = 32*(2*r-G*(T-1)) / (L^2*(T+1)^2)

The formula for the profile is now given by **y = r – ax^2**,[*]

Where “y” is the radius of the roller at a distance “x” from the roller’s midpoint.

[*]*Notice that the profile is parabolic, not elliptical, as some have thought.*