Drive Train Basics
(How to Be Sure Your Robot Will Turn)

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Background

A very common problem experienced by teams is that they put a lot of time into a four-wheel-drive system only to find out that it isn't able to turn. Every year there are countless questions on the FIRST forums asking for help with this subject. The purpose of this paper is to explain the physics behind the four-wheel-drive system, and to provide some guidelines that teams can use to help design their drive train so that it will turn.

One goal of this paper is to provide the detailed math to illustrate how this problem is solved. This is presented for the following reasons: a) to better understand the details of how the system works; b) to have an idea of how engineering principals can be used to solve a "mysterious" problem such as this; and c) to use as an exercise for training new team members to see if the problem can be solved independently and verified against this paper.

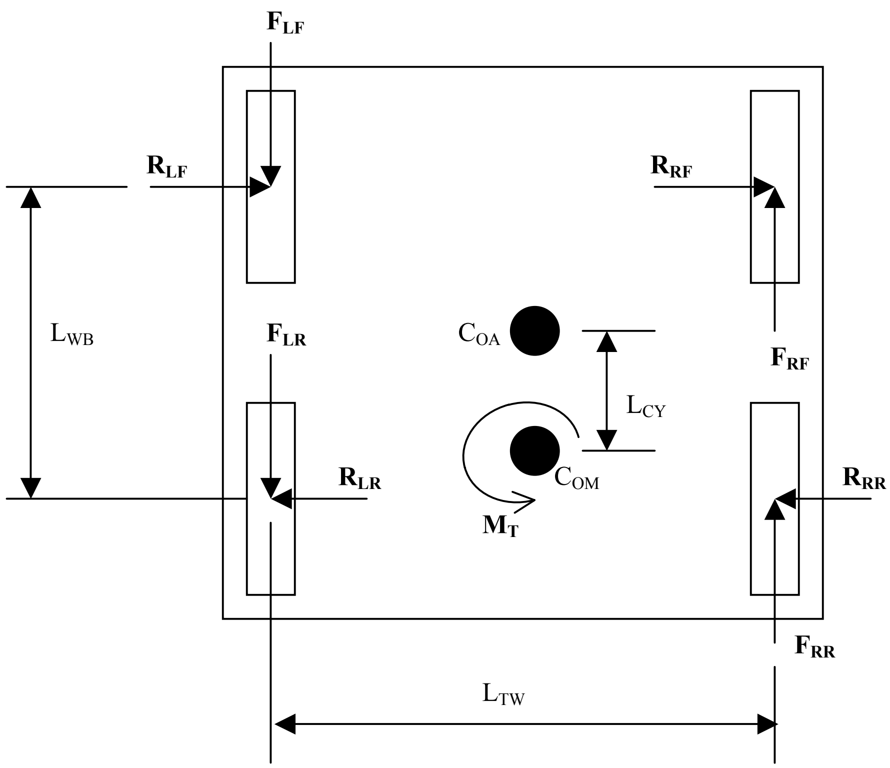
If the detailed math is not of interest, you may skip to the Conclusions and Extensions sections. These sections will contain some more qualitative guidelines and rules-of-thumb to help you with designing a drive train that will turn.

Details

When considering the physics of the system, one should keep in mind that this system is modeled with standard engineering practices. In other words, the physics aren't "exact". It is not the purpose to find an "exact" solution, since that would take much more work than is necessary, and is probably not possible to begin with. Therefore, some assumptions and simplifications are made (friction is a good example). As the punch line of the old engineering joke goes: "it isn't exact, but it's close enough for practical purposes."

A schematic drawing of a basic four-wheel-drive robot is shown below. The forces acting on each wheel are shown. $F_{??}$ is the force on each wheel that is provided by the motors, while $R_{??}$ is the reaction force (due to friction) from the robot attempting to turn. The subscript corresponds to the location: LF = left-front, RR = right-rear, etc.

Other notation:
L_{WB} = length of the wheel base
L_{TW} = length of the track width
COM = center of mass
COA = center of area; geometric center of the robot;
L_{CY} = distance from the center of area to the center of mass (note that it is assumed that the robot is left/right symmetric).
M_T = the resultant turning moment



$$C_{OA} = \left(\frac{L_{TW}}{2}, \frac{L_{WB}}{2} \right)$$

<u>Static Analysis</u>

We will be determining what is necessary to make the robot *just start* to turn. Therefore, this can be treated as a static (rather than dynamic) problem. It is common for rigid body dynamics problems to use the center of mass as the reference point of the body. Therefore, moments will be taken about the center of mass.

The first step will be to determine the moments about the center of mass. The contribution of each wheel is shown below (remember that Moment = Force X distance; in this case the distance is the *perpendicular* distance from the force to the center of mass).

$$M_{LR} = F_{LR}\frac{L_{TW}}{2} - R_{LR}\left(\frac{L_{WB}}{2} - L_{CY}\right)$$
$$M_{LF} = F_{LF}\frac{L_{TW}}{2} - R_{LF}\left(\frac{L_{WB}}{2} + L_{CY}\right)$$
$$M_{RF} = F_{RF}\frac{L_{TW}}{2} - R_{RF}\left(\frac{L_{WB}}{2} + L_{CY}\right)$$
$$M_{RR} = F_{RR}\frac{L_{TW}}{2} - R_{RR}\left(\frac{L_{WB}}{2} - L_{CY}\right)$$

We will assume that the gearing is low enough that the drive train is traction limited. See the section "Proper Gearing" to see how to make sure your drive is geared low enough to allow it to turn.

Assuming that the drive train is traction limited, the forces at each wheel will be the normal force at that wheel multiplied by the coefficient of friction between the wheel and the ground. Here, we assume that the wheel can have different coefficients of friction in the longitudinal (x) direction and the lateral (y) direction. Therefore the forces at each wheel can be written as follows:

$$F_{LR} = \mu_x N_R \qquad R_{LR} = \mu_y N_R$$
$$F_{LF} = \mu_x N_F \qquad R_{LF} = \mu_y N_F$$
$$F_{RF} = \mu_x N_F \qquad R_{RF} = \mu_y N_F$$
$$F_{RR} = \mu_x N_R \qquad R_{RR} = \mu_y N_R$$

In the above equations, N_R is the normal force on a rear wheel and N_F is the normal force on a front wheel.

Now, substitute the friction relationships into the wheel moment equations:

$$M_{LR} = \mu_x N_R \frac{L_{TW}}{2} - \mu_y N_R \left(\frac{L_{WB}}{2} - L_{CY} \right)$$
$$M_{LF} = \mu_x N_F \frac{L_{TW}}{2} - \mu_y N_F \left(\frac{L_{WB}}{2} + L_{CY} \right)$$
$$M_{RF} = \mu_x N_F \frac{L_{TW}}{2} - \mu_y N_F \left(\frac{L_{WB}}{2} + L_{CY} \right)$$
$$M_{RR} = \mu_x N_R \frac{L_{TW}}{2} - \mu_y N_R \left(\frac{L_{WB}}{2} - L_{CY} \right)$$

Now that the moment contribution of each wheel is known, sum the moments to find the resultant turning moment:

$$\begin{aligned} M_T = {} & \mu_x N_R \frac{L_{TW}}{2} + \mu_x N_F \frac{L_{TW}}{2} + \mu_x N_F \frac{L_{TW}}{2} + \mu_x N_R \frac{L_{TW}}{2} \\ & - \left[\mu_y N_R \left(\frac{L_{WB}}{2} - L_{CY} \right) + \mu_y N_F \left(\frac{L_{WB}}{2} + L_{CY} \right) + \mu_y N_F \left(\frac{L_{WB}}{2} + L_{CY} \right) + \mu_y N_R \left(\frac{L_{WB}}{2} - L_{CY} \right) \right] \end{aligned}$$

Collecting like terms, the moment equation can be rewritten as follows:

$$M_T = \frac{\mu_x L_{TW}}{2} \left(2N_R + 2N_F \right) - \mu_y \left[2N_R \left(\frac{L_{WB}}{2} - L_{CY} \right) + 2N_F \left(\frac{L_{WB}}{2} + L_{CY} \right) \right]$$

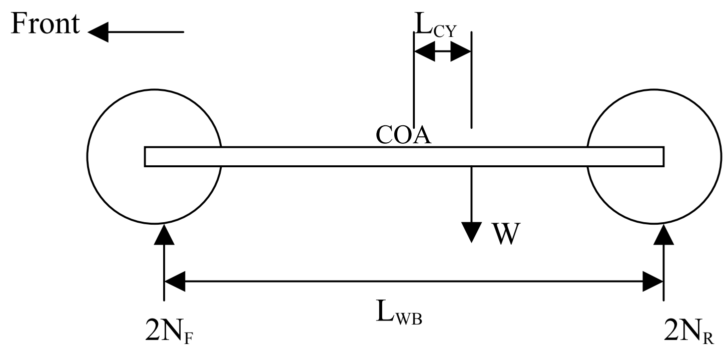
The first term can now be simplified using the distributive property to remove the factor of 2. The term in the brackets can also be simplified by using the distributive property.

$$M_T = \mu_x L_{TW} \left(N_R + N_F \right) - \mu_y \left[N_R L_{WB} - 2N_R L_{CY} + N_F L_{WB} + 2N_F L_{CY} \right]$$

Using the distributive property again on the term in the brackets, the above equation can be rearranged to get equation 1:

$$M_T = \mu_x L_{TW} \left(N_R + N_F \right) - \mu_y \left[L_{WB} \left(N_R + N_F \right) - 2L_{CY} \left(N_R - N_F \right) \right] \qquad \text{eqn(1)}$$

At this point, a relationship needs to be determined between the normal forces N_R and N_F, the robot weight W, and the location of the center of mass. The following picture is a side view of the robot. Notice that the force depicted on the front is 2 N_F. This is the case since there are 2 front wheels – not one. Likewise for the rear.



Using a static analysis again, take the sum of the forces in the z direction:

$$\sum F_z = 0:$$
$$2N_R + 2N_F - W = 0$$

Rearrange the above equation to get equation 2:

$$N_R + N_F = \frac{W}{2} \qquad \text{eqn (2)}$$

Now take the sum of the moments about the center of area:

$$\sum \mathrm{M_{COA}} = 0:$$
$$2N_R \frac{L_{WB}}{2} - 2N_F \frac{L_{WB}}{2} - WL_{CY} = 0$$
$$L_{WB}(N_R - N_F) = WL_{CY}$$

Rearrange the above equation to get equation 3:

$$N_R - N_F = W\frac{L_{CY}}{L_{WB}} \qquad \text{eqn (3)}$$

Recall equation 1:

$$M_T = \mu_x L_{TW}(N_R + N_F) - \mu_y\left[L_{WB}(N_R + N_F) - 2L_{CY}(N_R - N_F)\right] \qquad \text{eqn(1)}$$

Now, substitute equations 2 and 3 into equation 1 to get the following:

$$M_T = \mu_x L_{TW} \frac{W}{2} - \mu_y \left[L_{WB} \frac{W}{2} - 2L_{CY} W \frac{L_{CY}}{L_{WB}} \right]$$

Rearrange the above equation to get the following:

$$M_T = \frac{\mu_x L_{TW} W}{2} - \mu_y W \left[\frac{L_{WB}}{2} - 2\frac{L_{CY}^2}{L_{WB}} \right]$$

Lastly, rearrange further to get the final turning moment equation:

****** $$M_T = \frac{W}{2} \left[\mu_x L_{TW} - \mu_y \left(L_{WB} - 4\frac{L_{CY}^2}{L_{WB}} \right) \right]$$ ******

Making the Robot Turn

In order to make the robot turn, the final turning moment must be greater than zero. Therefore the following relationship must be true:

$$\mu_x L_{TW} > \mu_y \left(L_{WB} - 4\frac{L_{CY}^2}{L_{WB}} \right)$$ eqn (4) -- must be true for robot to turn!!!!

The relationship in equation 4 can be satisfied a number of ways. A few ways to satisfy the relationship are given below.

- You can move the COM toward the front or rear of the robot. As L_{CY} gets larger, the term in the parentheses will move toward a negative number. Thus, the right side of the inequality becomes smaller, allowing the left side to be greater than the right side; i.e. the relationship will be satisfied.

- Assume the worst-case COM: the COM is at the COA (i.e., $L_{CY} = 0$). Equation 4 then simplifies as follows:

$$\mu_x L_{TW} > \mu_y L_{WB}$$ eqn (5)

Now assume that you are using rubber wheels such that the coefficient of friction is the same in all directions. The friction coefficients in equation 5 then cancel. Then, the following relationship must be satisfied:

$$L_{TW} > L_{WB}$$

Therefore, by making the track width larger than your wheelbase, the robot will turn.

- Another method of satisfying equation 5 is to use a wheel with friction characteristics that are different in the x and y directions. For instance, you can use holonomic wheels. You can also use wheels that have a tread pattern that is rounded on the sides of the wheel, but sharp in the direction of travel. This way, the wheel bites into the carpet providing a high driving friction, but slides easily sideways.

Proper Gearing

To ensure that the robot will turn, the driving force generated by the wheels must be enough to overcome the frictional resistance force. To simplify things (and give some safety margin), we should select our gears such that the driving force at EACH wheel is enough to overcome the frictional resistance force at that wheel. Therefore, the moment about the COM at EACH wheel should be greater than zero.

In order to determine the frictional resistance at each wheel, we need to determine the normal force at each wheel. Recall equations 2 and 3:

$$N_R + N_F = \frac{W}{2} \qquad \text{eqn (2)}$$

$$N_R - N_F = W\frac{L_{CY}}{L_{WB}} \qquad \text{eqn (3)}$$

Notice that this is simply a system of two equations with two unknowns. Solve for the normal forces (left as an exercise for the reader):

$$N_R = \frac{W}{4} + \frac{W}{2}\frac{L_{CY}}{L_{WB}}$$

$$N_F = \frac{W}{4} - \frac{W}{2}\frac{L_{CY}}{L_{WB}}$$

This shows that if Lcy is positive, the normal force will be greater at the rear of the robot. This should make sense since if Lcy is positive, the center of mass is toward the rear of the robot, which means that there should be more normal force on the rear wheels.

Therefore, to be sure that the worst case is used, the analysis will be performed for a rear wheel (Lcy is assumed to be positive for this example).

To ensure turning, the moment about the COM for the left-rear wheel must be greater than zero:

$$F_{LR}\frac{L_{TW}}{2} - \mu_y N_R\left(\frac{L_{WB}}{2} - L_{CY}\right) > 0$$

Substitute in the equation for the rear normal force:

$$F_{LR}\frac{L_{TW}}{2} - \mu_y\left(\frac{W}{4} + \frac{W}{2}\frac{L_{CY}}{L_{WB}}\right)\left(\frac{L_{WB}}{2} - L_{CY}\right) > 0$$

Rearrange and multiply out the two terms in parentheses:

$$F_{LR}\frac{L_{TW}}{2} > \mu_y\left(\frac{WL_{WB}}{8} - \frac{WL_{CY}}{4} + \frac{WL_{CY}}{4} - \frac{WL_{CY}^2}{2L_{WB}}\right)$$

Simplifying:

$$F_{LR}\frac{L_{TW}}{2} > \frac{\mu_y W}{2}\left(\frac{L_{WB}}{4} - \frac{L_{CY}^2}{L_{WB}}\right)$$

Lastly, solve for the force:

****** $F_{LR} > \frac{\mu_y W}{L_{TW}}\left(\frac{L_{WB}}{4} - \frac{L_{CY}^2}{L_{WB}}\right)$ ****** This relationship must be satisfied in order to turn

Therefore, you can use the above equation to determine the minimum force necessary at the wheels to make the robot turn. Using this force, along with the motor characteristics and the diameter of your wheels, you can determine the gear ratio that will satisfy this minimum force. Of course, you should use a gear ratio that provides *considerably more* than this minimum force, since this force is just *barely* enough to make the robot start to turn.

Extensions

Tank Treads

The fundamental difference between tank treads and 4-wheel-drive is that the force on the treads is distributed over the length of the treads, rather than at four discrete points. In order to take the effects fully into account, the forces and moments need to be integrated over the length of the tread.

The details of the math will be skipped for this, jumping straight to the conclusions instead.

The good news is that the 4-wheel-drive equations can still be used. The track-width (L_{TW}) is still the distance between the two treads. However, the wheelbase (L_{WB}) is not simply the length of the contact patch of the tread. Instead, L_{WB} is 2/3 of the length of the contact patch of the treads. Other than this, the equations can still be used.

Conclusions

1) When designing a drive train, begin by being sure that the robot will turn. To do this, be sure that the following relationship is true:

$$\mu_x L_{TW} > \mu_y \left(L_{WB} - 4\frac{L_{CY}^2}{L_{WB}} \right)$$

2) If you don't like math, use the following rules of thumb:
   - Make the track width greater than the wheel base ($L_{TW} > L_{WB}$)
   - If possible, reduce the lateral friction coefficient while keeping the longitudinal friction high (i.e., use holonomic wheels or choose a good wheel tread pattern).
   - Try to move the center of mass slightly away from the center of the robot. Use caution to not move the COM far enough so that the robot becomes unstable.

3) Once you satisfy the equation in step 1), you should have values for the important dimensions (L_{TW}, L_{WB}, and L_{CY}). Plug these numbers into the following equation to determine the force necessary at each wheel to make the robot turn:

$$F_{LR} > \frac{\mu_y W}{L_{TW}} \left(\frac{L_{WB}}{4} - \frac{L_{CY}^2}{L_{WB}} \right)$$

4) Using the force calculated in step 3), a minimum gearing can be determined such that the motors can produce the necessary force at the wheel.

Keep in mind that these equations are generated such that the robot will *just barely begin* to turn. When you are creating your design, some safety margin should be included into the calculations. For instance, if you determine in step 3 that you need to produce 50 lb of force at each wheel, you might consider that your gearing should produce 100 lb of force at the wheels, just to be safe. I would suggest a safety factor of at least 2 for the gearing.

You should also keep in mind that your drive train and gearing will not be 100% efficient. Therefore, when determining your gearing, be sure to take efficiency into account.

Final Conclusion

The physics used to develop these equations and relationships are not exact. However, by carefully making a few simplifying assumptions, the problem can become easy enough to solve, and useful results can be achieved. By using the above equations and rules of thumb, you should be able to be successful in designing a four-wheel-drive system.