DC Motors and Gearboxes

AKA, “Real motors have curves!” and “Selecting a criterion is HARD”

What is in a gearbox and why do we use one?

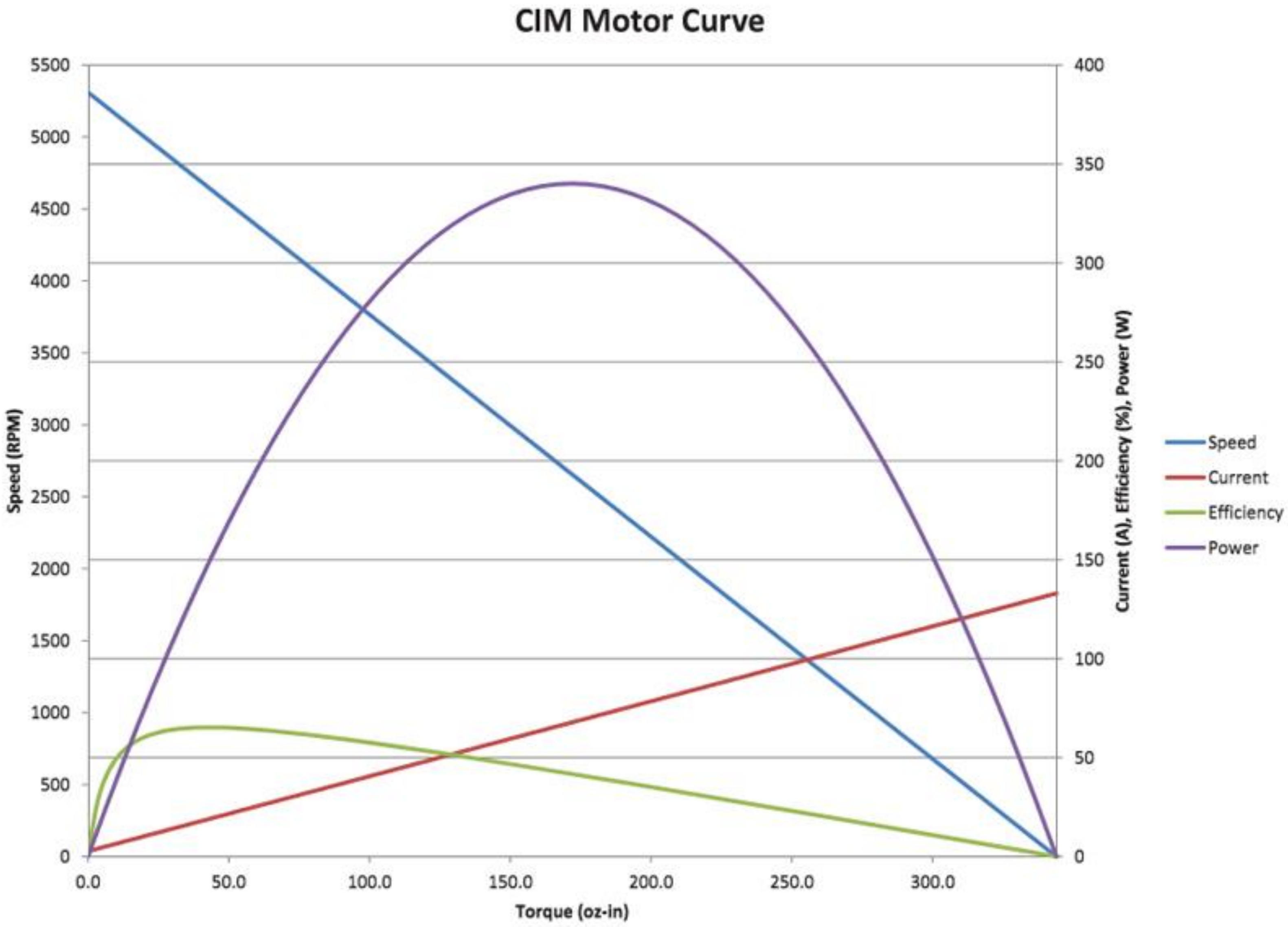
- Motors
- Gears mated together
- Chain and belting may also be employed (see lower right)
- Usually DC motors run at high speeds with relatively low torques- we want to lower their speed, or at least increase their torque. (Sometimes you might want to increase speed)
- Many different types of gears

What Properties do motors have?

- Torque. Motors cannot produce force by themselves, only torques. Torque is measured in force-distance (I.E. newton-meters (N*m or N-m) or ft-lb or oz-in). Torque = lever arm distance × force. Torque may be seen as the greek letter tau. $\tau = r \times F$
- Angular Velocity. Motors don't have a linear speed, either. They have a speed typically measured in RPM (though, a more mathematically useful unit is rad/s). It is represented as greek letter omega. Angular velocity × radius = velocity. $\omega \times r = v$
- "Stall" or max torque- the maximum torque that a motor can apply. (Stall is when the motor is not spinning)
- Free speed- the maximum angular velocity the motor will obtain if no load is applied.
- Stall current- motors will consume current most at stall and will decrease as angular velocity increases.

Motors Have Curves

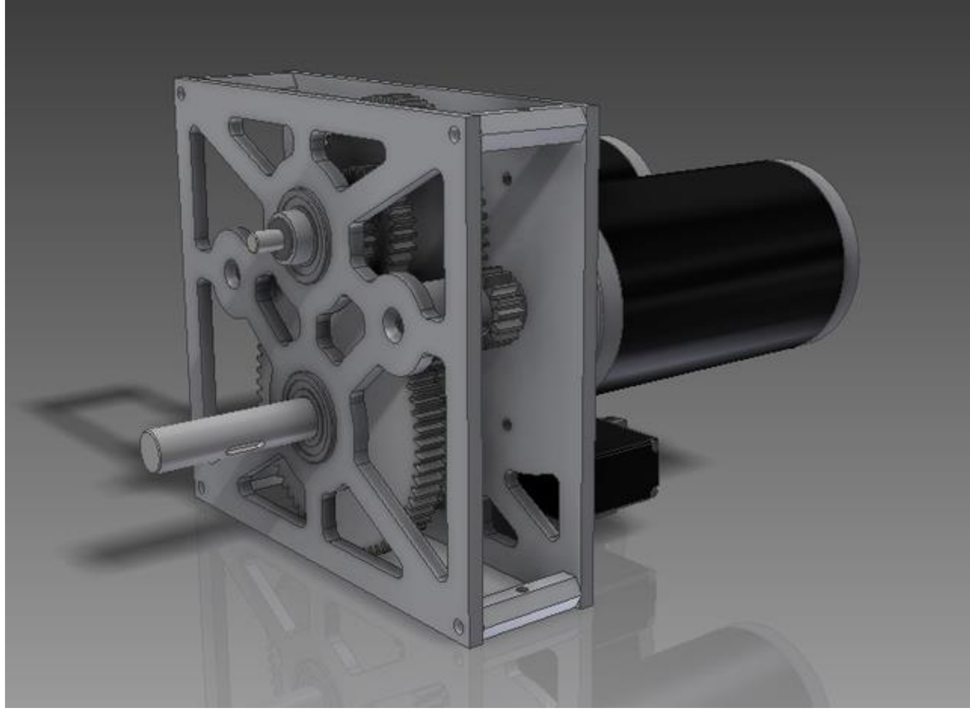
- Graph on right shows the relationship that angular velocity (speed) has on torque. The higher the speed, the lower the torque.
- This also means peak power is obtained at about 50% free speed. (Power = torque * angular velocity)



Gearbox terminology

- Spur gear- a typical gear.
- Pinion- The gear attached to the motor.
- Cluster gear- One of the gears in the middle of the gearbox ratio. There are two of these- a large and a small cluster gear.
- Output gear- The final gear in the gearbox.
- Idler gear- A gear on a geartrain that isn’t driven by the shaft or an output; it only exists to transmit power from one gear to another gear. It does not effect gear ratio.

Gearbox Ratios



- The most common gearbox is a two stage, spur gearbox.
- The “gear ratio” of a gearbox is a measure of how much torque is increased / speed is decreased. I.E. a gearbox of ratio 12:1 will increase torque 12 times and decrease speed 12 times (and 12 turns on the input will result in 1 turn on the output).
- Gear ratio of two spur gears is simple: Divide the input teeth by the output teeth. A 12 tooth gear that drives a 48 tooth gear is a ratio of 4:1 (or 4)
- Gear ratio of multiple stages is also simple: Multiply the ratios. If you have a 2:1 ratio driving a 4:1 ratio, the overall ratio is 8:1.

Motors (and thus gearboxes)… have equations.

```
> restart
```

Acceleration is derivative of position

```
> a:=diff(v(t), t)
```

$$a := \frac{d}{dt} v(t)$$

Force exerted by the wheel is the output torque of the gearbox divided by the wheel radius

```
> Fwheel:=torqueout/r
```

$$Fwheel := \frac{torqueout}{r}$$

Torque output of the gearbox is governed by the linear relationship of the motor curve, all multiplied by the gear ratio of the gearbox.

```
> torqueout:=(torquemax-(omega/omegamax)*torquemax)*g
```

$$torqueout := \left(torquemax - \frac{\omega\ torquemax}{omegamax}\right) g$$

This governs torque output of a gearbox as a function of max motor torque, instantaneous angular velocity, max motor speed, and gear ratio.

The angular velocity of the motor is the angular velocity of the wheel (v/r) times the gear ratio.

```
> omega:=v(t)/r*g
```

$$\omega := \frac{v(t)\ g}{r}$$

Solve the differential equation: F = m*a (with initial condition v(0)=0

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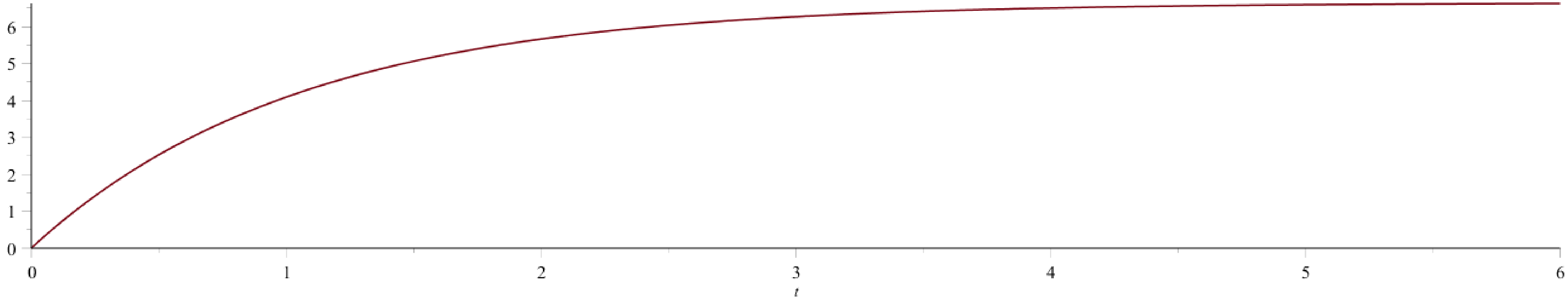
> v:=rhs(dsolve({Fwheel = m*a, v(0)=0}))
```

$$v := \frac{r\ omegamax}{g} - \frac{e^{-\frac{g^2\ torquemax\ t}{r^2\ omegamax\ m}}\ r\ omegamax}{g}$$

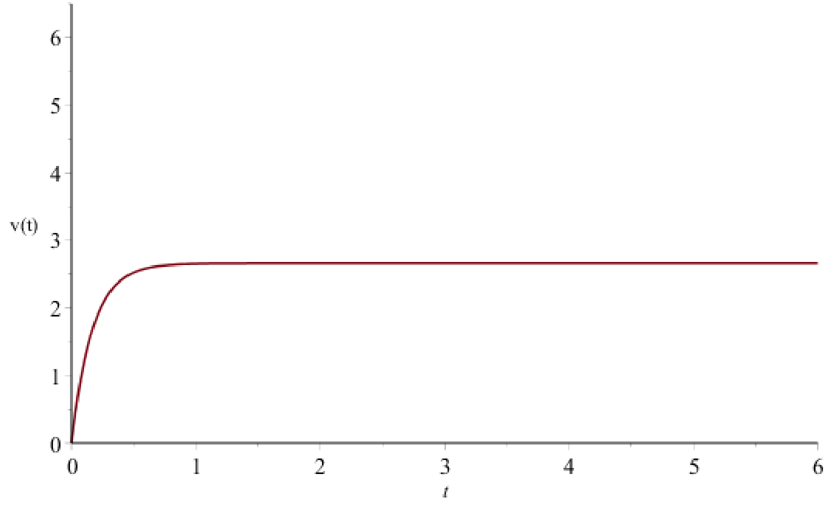
This is velocity as a function of time… With max motor speed, gear ratio, wheel radius, robot mass, and max torque as constants.

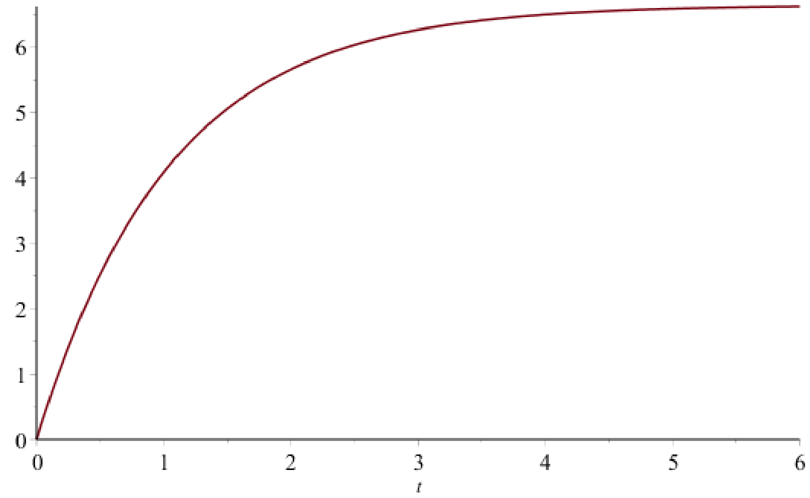
```
> conditions:=[
    torquemax=2.42*2, # Combined maximum torque that the motor(s) can supply before the gear ratio you wish to optimize, in Newton-meters
    omegamax=5000/60*2*Pi,    # Maximum free speed of the motor, where torque = 0, in rad/sec, NOT RPM!
    r=4*2.54/100/2,       # Radius of the wheel, in meters
    m=60,              # Mass of the vehicle in kg
    g=4
  ]:
> plot(subs(conditions, v), t=0..6)
```



Yes, it is possible to optimize the gear ratio using calculus. But more generally, here's the facts about velocity of a bot powered by DC motor:

- Velocity obeys a curve of type (1-e^t) (You'll run into these a lot in physics when modeling with differential equations.). The robot will initially sharply accelerate but acceleration decreases as time goes on.
- Lower (as in a 1:10) gear ratios (top v vs t graph) are better for "sprinting". They allow you to quickly accelerate, but the maximum speeds you can obtain are lower. (Top image)
- Higher (as in a 1:4) gear ratios are better for long, uninterrupted runs. You can't accelerate as quickly, but the maximum speeds are higher. (Lower Image)





BUT WAIT, THERE'S MORE!

- Those simplified equations don't account for:
- Friction. There is indeed friction, especially during turns.
- Extra loads. If you need to go up a hump, that requires even more force.
- Power brownouts. CIM motors draw a lot of current. It is possible to trip the main breaker! When you pull a lot of current (I.E. a 6 CIM drivetrain at stall), you'll actually have less power output than in actuality.
- Motor strain. You should select the right motor for each position (see next slide)