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Differential Equations of Motion for Frisbee: Derivation and Numerical Solution

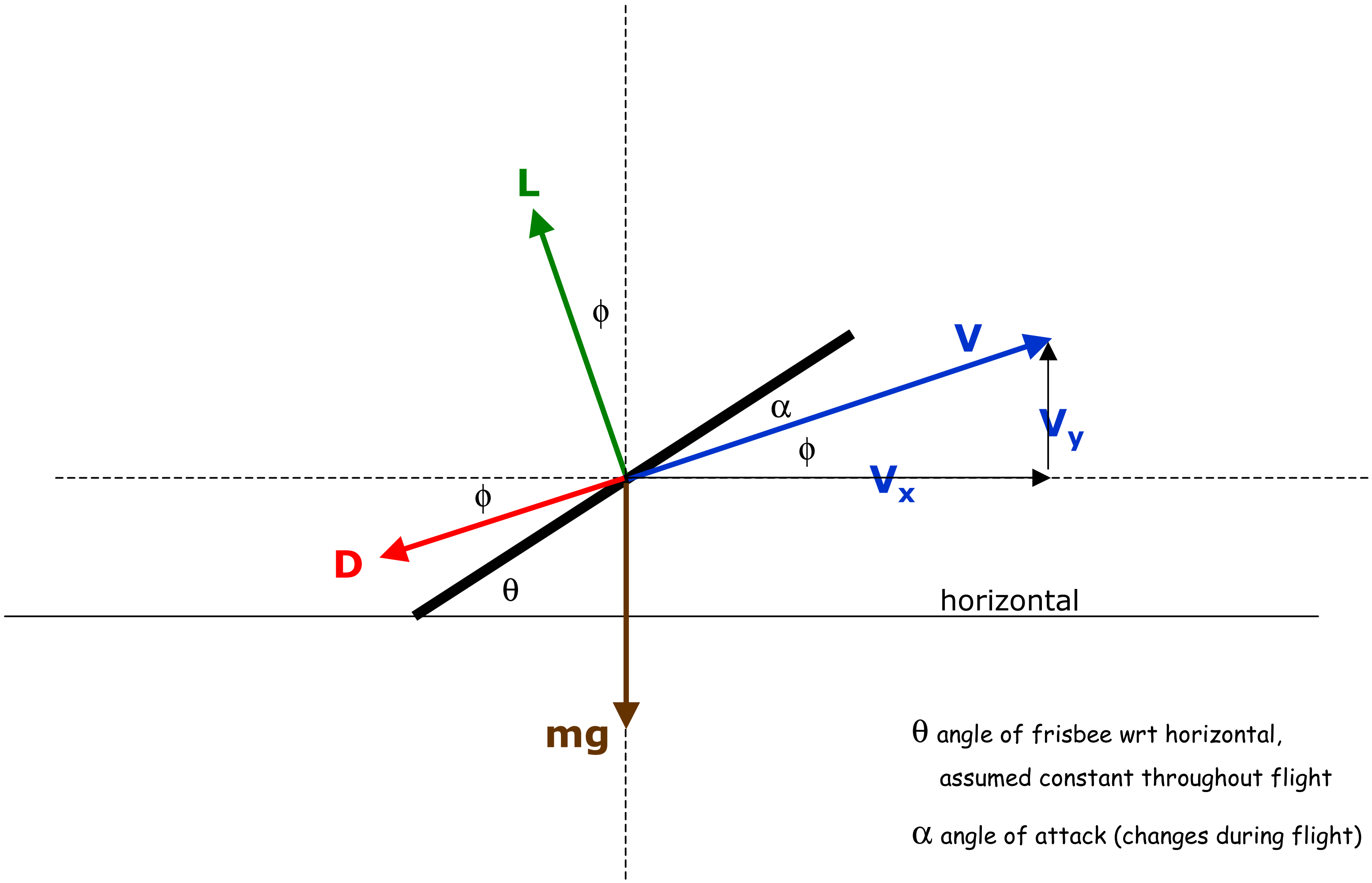

Figure 1

Refer to Figure 1 on previous page

$$\text{computeAccelerations}(Vx, Vy, ax, ay) = \{$$

$$V^2 = \sqrt{Vx^2 + Vy^2}$$

$$\phi = \operatorname{atan}\left(\frac{Vy}{Vx}\right)$$

$$\alpha = \theta - \phi$$

$$C_L = C_{L_o} + C_{L_\alpha}\,\alpha$$

$$C_D = C_{D_o} + C_{D_\alpha}(\alpha - \alpha_0)^2$$

$$D = \frac{1}{2}\,C_D\,\rho\,A\,V^2$$

$$L = \frac{1}{2}\,C_L\,\rho\,A\,V^2$$

$$Fx = -D\cos(\phi) - L\sin(\phi)$$

$$Fy = L\cos(\phi) - D\sin(\phi) - m\,g$$

$$ax = \frac{Fx}{m}$$

$$ay = \frac{Fy}{m}$$

$$\}$$

NUMERICAL SOLUTION

Given:

computeAccelerations() function from previous page

Assume frisbee does not roll or pitch during flight (i.e assume only forces are Lift, Drag, and Gravity all acting in the vertical plane of the flight path)

…And the launch parameters:

Assume that θ is equal to the launch angle and does not change during flight

Assume that the launch speed, Vo, is in the same direction as launch angle, i.e. $\phi_o = \theta$

…And the initial conditions:

Vx = Vo cos(θ) = initial x-velocity

Vy = Vo sin(θ) = initial y-velocity

X = Xo

Y = Yo

(continued on next page)

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Numerical simulation using Heun's Method:

```
ax(Vx,Vy,Ax,Ay)      // compute Ax & Ay

Vxn = Vx + Ax*dt
Vyn = Vy + Ay*dt

ax(Vxn,Vyn,Axn,Ayn)  // compute Axn & Ayn

Vxn = Vx + (Ax+Axn)/2*dt
Vyn = Vy + (Ay+Ayn)/2*dt

X = X + (Vx+Vxn)/2*dt
Y = Y + (Vy+Vyn)/2*dt

Vx = Vxn
Vy = Vyn

(repeat)
```

References

Frisbee Flight Simulation and Throw Biomechanics

http://biosport.ucdavis.edu/research-projects/frisbee-flight-simulation-and-throw-biomechanics/HummelThesis.pdf

http://morleyfielddgc.files.wordpress.com/2009/04/hummelthesis.pdf

Heun's Method

http://math.fullerton.edu/mathews/n2003/Heun%27sMethodMod.html