Scenario6:

Problem Statement:

4-wheel skid-steer with front and rear wheels chained together on each side; torque is naturally distributed to the wheel with higher max static friction once static friction limit has been reached at the other wheel. [see endnote 1]

Maximum coefficient of friction occurs in the Y direction. Minimum coefficient of static friction occurs in the X direction. For any other direction, elliptical interpolation between uy and ux is used to compute the effective static coefficient in that direction. *The static coefficients for the front wheels are not necessarily the same as for the rear.*

Rectangular wheel pattern, Lwb<>Ltw

CoM is aft of CoA

Equal and opposite gearbox torque is being applied on left and right sides.

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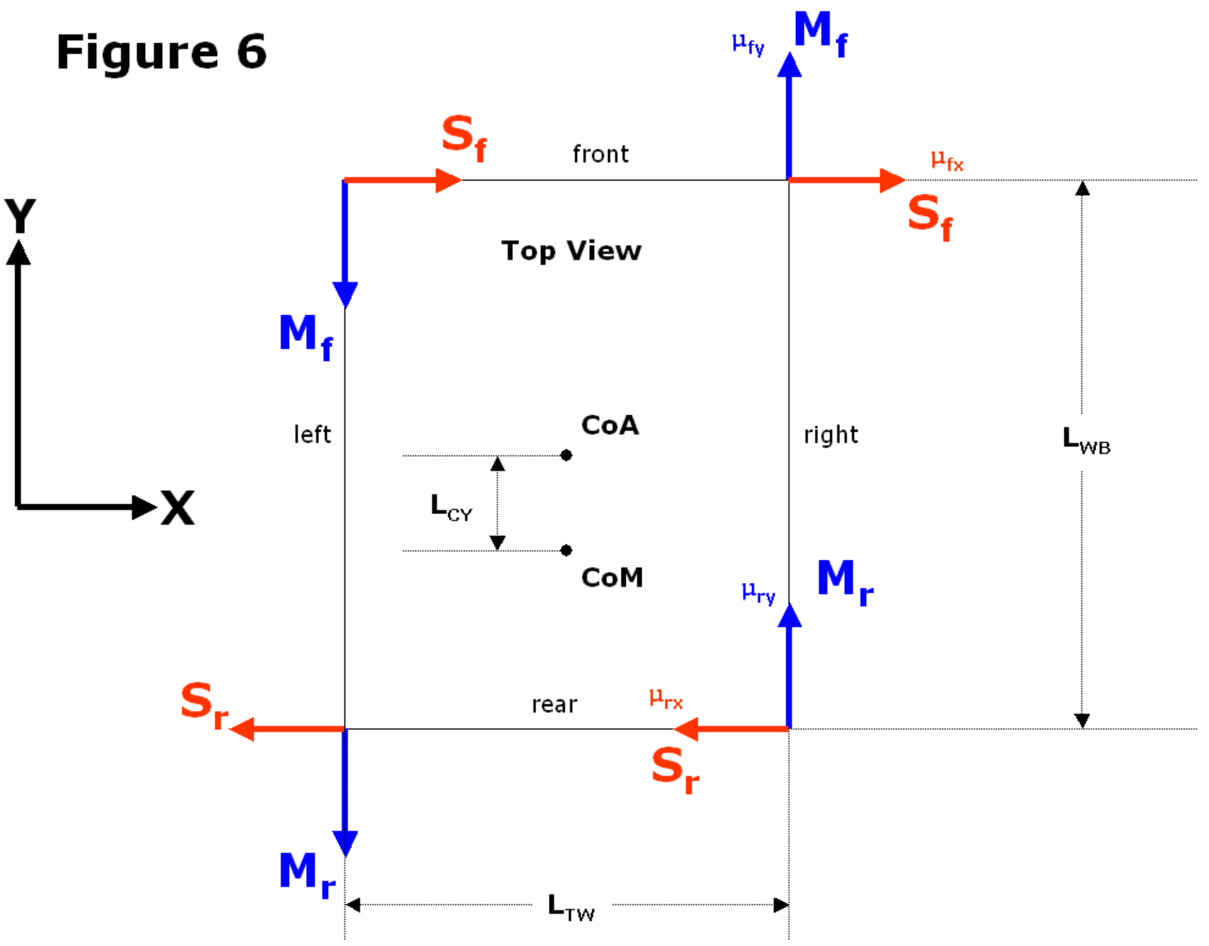
Figure 6



Solution:

For this analysis, when the vehicle is in static equilibrium with the maximum sustainable (sustainable by static friction, that is) gearbox torque (equal and opposite gearbox torque on each side) applied to the chains on each side, the gearbox torque at this static condition represents the torque above which static equilibrium can no longer be sustained and the vehicle will start to move. It is the solution to the analysis.

Refer to Figure6.

The goal is to find the maximum sum (Mf+Mr) which can be sustained by the available static friction of the wheels on the floor.

By symmetry, the two Sf forces are equal, and the two Sr forces are equal. By force balance in the X direction, Sf=Sr=S. All four S forces are equal.

To remain in static equilibrium, the net torque on the vehicle must be zero around any point. Choose the right front wheel for convenience. Then:

$$(M_f + M_r) * L_{tw} = 2 * S * L_{wb}$$

$$S = \frac{1}{2} * (M_f + M_r) * \frac{L_{tw}}{L_{wb}}$$

$$S = \frac{1}{2} * (M_f + M_r) * f_2$$

where f2 is defined as

$$f_2 = \frac{L_{tw}}{L_{wb}}$$

Let Ff be the net friction force on each front wheel. Then

$$F_f = \sqrt{S^2 + M_f^2}$$

Let uf be the effective coefficient of static friction on each front wheel. Then, using elliptical interpolation between ufy and ufx:

$$\mu_f = \frac{F_f}{\sqrt{\left(\frac{S}{\mu_{fx}}\right)^2 + \left(\frac{M_f}{\mu_{fy}}\right)^2}}$$

Let W be the vehicle weight.

Let Nf be the normal force on each front wheel.   Define f_1 as:

$$f_1 = \frac{L_{cy}}{\frac{L_{wb}}{2}}$$

Then

$$N_f = \frac{W}{4} * (1 - f_1)$$

[see endnote2]

...and this constraint must be satisfied for the vehicle to be in static equilibrium:

$$\mu_f * N_f - F_f > 0$$

Let Fr be the net friction force on each rear wheel. Then

$$F_r = \sqrt{S^2 + M_r^2}$$

Let ur be the effective coefficient of static friction on each rear wheel. Then, using elliptical interpolation between ury and urx:

$$\mu_r = \frac{F_r}{\sqrt{\left(\frac{S}{\mu_{rx}}\right)^2 + \left(\frac{M_r}{\mu_{ry}}\right)^2}}$$

Let Nr be the normal force on each rear wheel. Then

$$N_r = \frac{W}{4} * (1 + f_1)$$

[see endnote 2]

... and this constraint must be satisfied for the vehicle to be in static equilibrium:

$$\mu_r * N_r - F_r > 0$$

Two more constraints are added to prevent anomalous solutions:

$$M_f >= 0$$

$$M_r >= 0$$

Everything is now set up to perform the nonlinear constrained optimization to maximize (Mf+Mr) subject to the 4 constraints by varying Mf and Mr. Each of the four constraints is a function of Mf, Mr, and the eight numerical values given by the user for the design parameters W, Ltw, Lwb, Lcy, ufx, ufy, urx& ury.

See the Excel and Maxima solution files.

End Notes:

[1] The front/rear torque distribution is pivotal in doing a real-world analysis of the limiting static equilibrium in a robot with front/rear wheels chained together on each side.

As gearbox torque is increased to the point where the front (or rear) wheels reach their limiting static friction, any further gearbox torque will go to the other wheels, until those wheels also reach their limiting static friction, at which point static equilibrium can no longer be maintained (and the robot will turn).

As the applied gearbox torque is being increased to this maximum sustainable value, there may be some imperceptible rotation of the front (or rear) wheels once they reach their maximum static friction as they transfer chain tension from front to rear (or vice-versa). Assuming the chain is tensioned properly, it is assumed that this minor and momentary rotation will not be sufficient to de-stablize the static equilibrium.

[2] This follows from torque and force balance, in the vertical plane, of the force of gravity (acting at the center of mass) and the normal forces (acting at the wheels).