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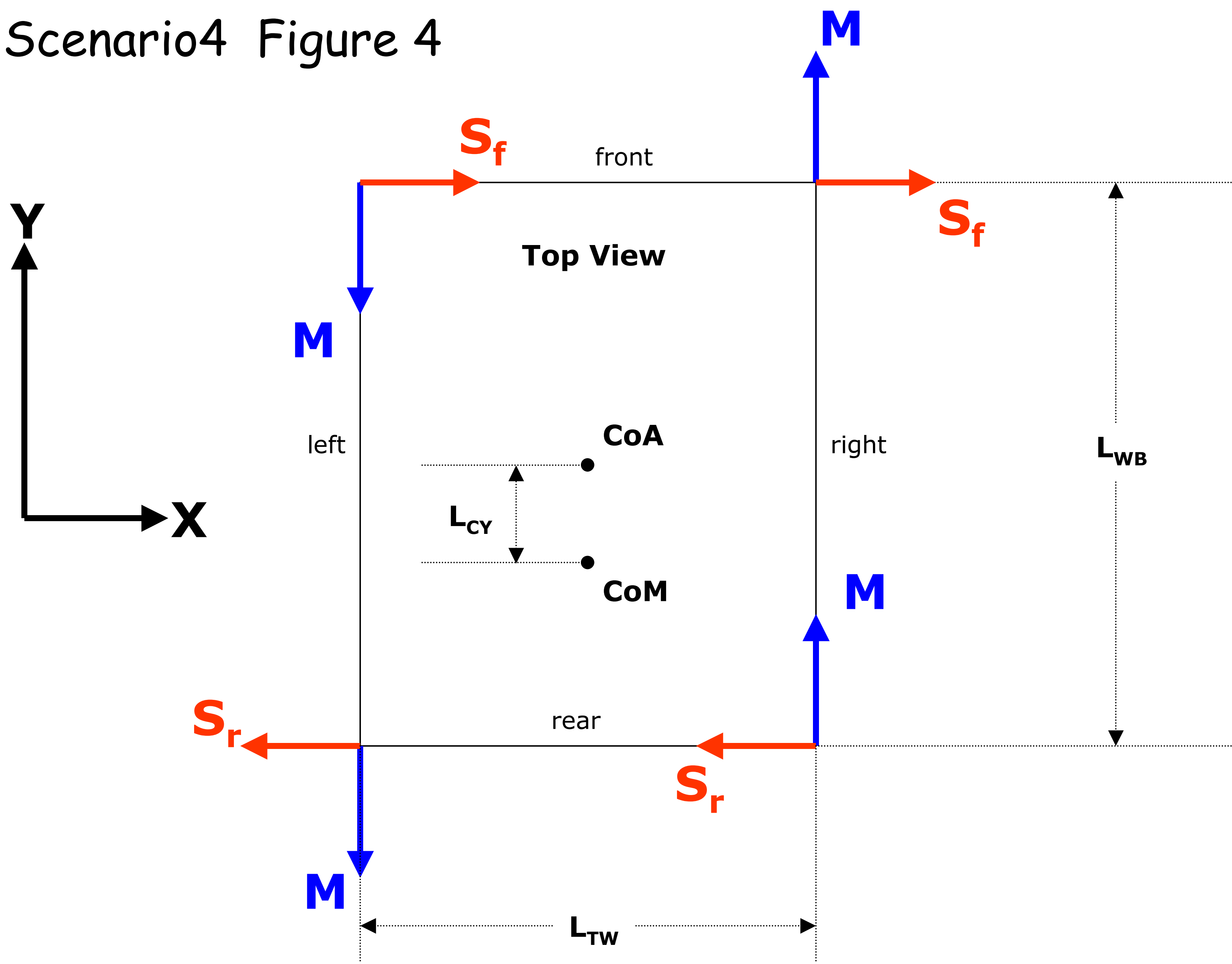
Scenario4 Figure 4



See Figure 4 on previous page.

- rectangular (non-square) wheel pattern (trackwidth<>wheelbase)
- all four wheels identical
- CoM located aft of CoA
- Maximum coefficient of static friction μ_y occurs in Y direction. Minimum μ_x occurs in X direction. Off-axis μ assumed to be an elliptical interpolation between μ_x and μ_y
- assume each wheel is being driven with the same torque magnitude (requires independent drive of all 4 wheels; front/rear wheels NOT chained together on each side)

The vectors M and S on the diagram are the Y and X components, respectively, of the net force that the carpet is exerting in the XY plane on the bottom of each wheel.

The magnitude of M is equal to the driving torque being applied to the wheel divided by the radius of the wheel.

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Proceed as follows:

All four M forces are equal to each other in magnitude (it is stipulated in the problem statement that all four wheels are being independently driven with the same torque magnitude)

By symmetry, the two front Sf forces are equal to each other, and the two rear Sr forces are equal to each other. By force balance in the X direction, Sf = Sr = S.

To be in equilibrium, the net torque acting on the vehicle must be zero around any point. To simplify the math, select the front right wheel as the point.
Then $2*S*L_{WB} = 2*M*L_{TW}$ => $S=M*(L_{TW}/L_{WB})$. Define $f_2 = L_{TW}/L_{WB}$ => $S=f_2*M$

The magnitude of the net force on each wheel is $F=\text{sqrt}(M^2+S^2)=M*\text{sqrt}(1+f_2^2)$

Because the CoM is aft of the CoA, the normal force at the front and rear wheels is given by $N_f=(W/4)*(1-f_1)$ and $N_r=(W/4)*(1+f_1)$, respectively, where f_1 is defined as $L_{CY}/(L_{WB}/2)$. W is the vehicle weight. [see endnote1]

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The maximum available static friction at the front wheels is less than the rear, because Nf<Nr. Thus when F > mu*Nf , static friction can no longer sustain vehicle equilibrium, and the vehicle will begin to turn [see endnote2]:

$F > mu*Nf \Rightarrow M*sqrt(1+f_2^2) > mu*(W/4)*(1-f_1) \Rightarrow$

$M > mu*(W/4)*(1-f_1)/sqrt(1+f_2^2)$

In the above inequality, mu is given by the elliptical interpolation:

$mu=F/sqrt((Fx/u_x)^2+(Fy/u_y)^2)$

… where F is the net friction and Fx, Fy are its X,Y components. Therefore:

$M > (W/4)*(1-f_1)/sqrt(1/u_y^2 +(f_2/u_x)^2)$ [see endnote3]

To turn, the torque on each wheel must be greater than radius*M

Note: when $u_x=u_y$, the above reduces to the solution for Scenario3

EndNotes:

[1] This follows directly from force and torque balance in the vertical plane passing through the vehicle's longitudinal axis.

[2] When the front wheels break traction they will start to spin, since they are not chained to the rear. When this occurs, the friction on the front wheels will be kinetic. Kinetic friction always acts in the direction of the relative motion of the two interacting surfaces (in this case, the front wheels and the floor). If the vehicle is not yet rotating, the kinetic friction on the front wheels will be aligned along the Y axis, and there will be no friction component in the X direction (i.e., Sf=0). Thus there will be a net non-zero torque on the vehicle, causing it to start rotating. When the vehicle's rotational speed becomes non-zero, there *will be* a sideways component to the kinetic friction, because the relative motion between the wheel and the floor is no longer aligned along the vehicle's Y axis. The rotational speed will increase until this sideways component becomes sufficient to balance the net torque on the vehicle.

[3] Substitute the expression for mu and simplify. This could be shown in a footnote; it is straightforward algebra.