Vertical Trajectory

Objects moving at high speeds through air encounter air drag proportional to the square of the velocity. Describing the motion of objects under this quadratic drag usually requires numerical techniques rather than straight analytic formuli since the drag force and the gravitational force are not acting along the same line. The case of the vertical trajectory can be treated analytically since the forces are colinear. It is common practice to express the velocity and time in terms of the terminal velocity v_t and a characteristic time τ.

$$t_{peak} = \tau \tan^{-1}\left[\frac{v_0}{v_t}\right]$$

$$y_{peak} = -v_t \tau \ln\cos\left[\tan^{-1}\frac{v_0}{v_t}\right]$$

v_t = terminal velocity

τ = characteristic time

At height y on the way down:

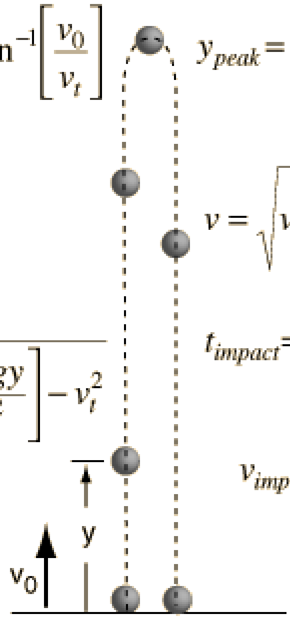
$$v = \sqrt{v_t^2 - v_t^2 \exp\left[\frac{-2g(y_{peak}-y)}{v_t^2}\right]}$$

At height y on the way up:

$$v = \sqrt{\left(v_t^2 + v_0^2\right)\exp\left[\frac{-2gy}{v_t^2}\right] - v_t^2}$$

$$t_{impact} = \tau \cosh^{-1}\left[\exp\left(\frac{y_{peak}}{v_t \tau}\right)\right]$$

$$v_{impact} = v_t \sqrt{1 - \exp\left[\frac{-2g y_{peak}}{v_t^2}\right]} = \frac{v_t v_0}{\sqrt{v_t^2 + v_0^2}}$$



Two common approaches to the quadratic drag force are:

1. Express the drag in terms of a single drag coefficient c: $f_{drag} = -cv^2$	2. Assume that drag is proportional to cross-sectional area and express in terms of area and a shape-dependent drag coefficient C: $f_{drag} = -\frac{1}{2}C\rho * Av^2$

Index

Fluid friction

HyperPhysics***** Mechanics ***** Fluids *R Nave*

Go Back

Vertical Trajectory Calculation

Air drag can be expressed in the form

$$f_{drag} = -\frac{1}{2} C\rho * Av^2$$

and it is equal to the weight of the object at the terminal velocity

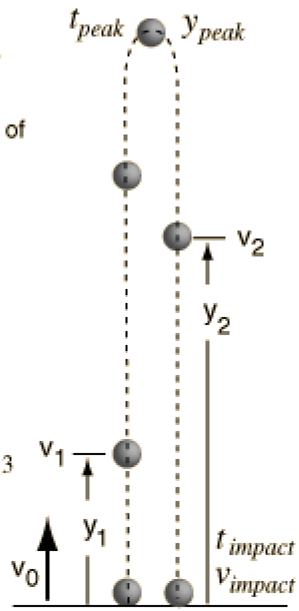
$$v_t = \sqrt{\frac{2mg}{C\rho * A}}$$

The characteristic time is

$$\tau = \frac{v_t}{g}$$

ρ^* = air density = 1.29 kg/m^3 nominally.

C = 0.5 for a sphere



For a sphere of radius
r = 8.89 cm
and density
ρ = 77.31200 kg/m^3
ρ = 0.077312 gm/cm^3

the mass is
m = 227.5311 gm.

Assume the object has a drag coefficient C = 0.5 (the default value is C = 0.5 for a sphere) and is falling through air with density ρ^* = 1.29 kg/m^3 (the default value is 1.29 kg/m^3).

Under these conditions, it's terminal velocity will be
v_t = 16.68752 m/s
v_t = 60.07508 km/hr
v_t = 37.32832 mi/hr

and its characteristic time is
τ = 1.702808 s.

Index

Fluid friction

If this sphere is launched vertically with a velocity of
v_0 = 12 m/s = 39.37007 ft/s

then on the way up at height y_1 = 3.048 m = 10 ft
it will have velocity v_1 = 7.901215 m/s = 25.92262 ft/s

It will reach a peak height y_{peak} = 5.921878 m = 19.42873 ft
at time t_{peak} = 1.061582 s

On the way down at height y_2 = 1.524 m = 5 ft
it will have velocity v_2 = 8.610097 m/s = 28.24835 ft/s

It will reach the ground at velocity
v_{impact} = 9.742567 m/s = 31.96380 ft/s = 35.07324 km/hr = 21.79314 mi/hr
at time t_{impact} = 1.137891 s.

For comparison, if there were no air friction the projectile would have reached height

h = 7.346938 m = 24.10412 ft

and would impact with the original launch velocity at time t = 2.448979 s.

HyperPhysics***** Mechanics ***** Fluids *R Nave* Go Back