Design of a Catapult Utilizing Torsion Springs for "Aerial Assault"

Ralph Carl

Mentor Team 20 Rocketeers

2014

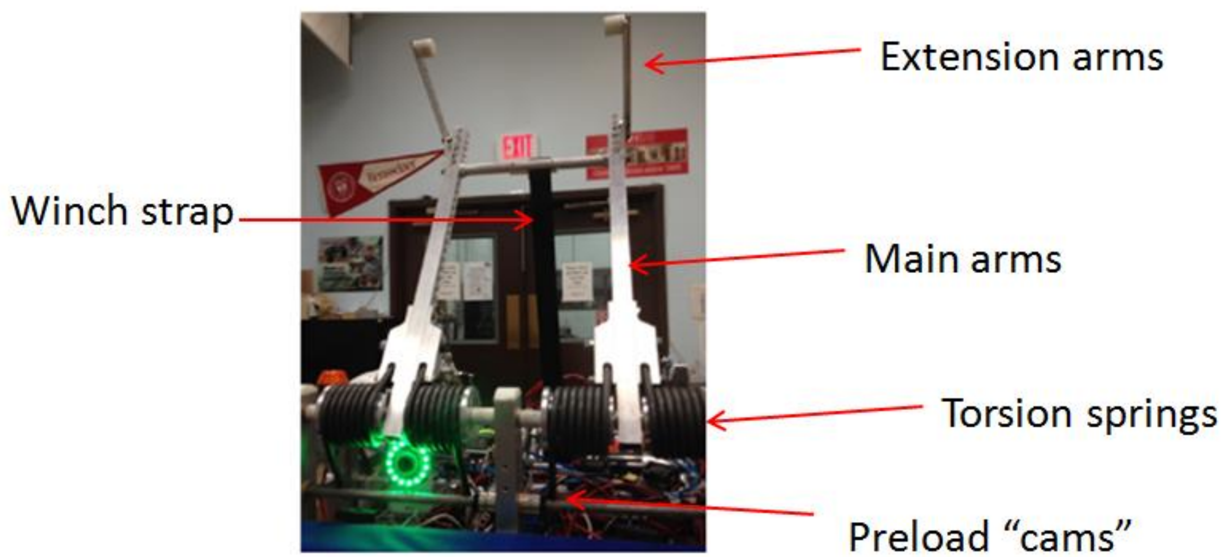
Abstract

The calculations for the sizing of the springs and ball trajectories for a torsion spring catapult used by the Team 20 Eclipse robot are described

Introduction

Torsion springs are used as the source of energy which moves an arm which launches the ball on its trajectory. A photograph of the catapult is shown in Figure 1.

Figure 1. Photograph of catapult on the Team 20 Eclipse Robot



Design Objectives

The catapult nominally has three target operating modes (trajectories) as follows:

1) Score in the high goal from a broad range of distances (This is the same as for launching over the truss.)

2) Pass the ball all the way across the field to a "hail Mary" receiver

3) Expel the ball through the "collector" for a low pass or to score on the long goal.

The catapult springs and arm are sized for sufficient strength to be able to make the full court pass with the springs highly preloaded. When the springs have a smaller preload, the catapult is configured to make a shot. To expel the ball the catapult is discharged with the motors engaged to slowly release the catapult.

To score in the high goal from the widest range of field positions the ball trajectory is such as to maximize the distance at which the ball is at a scoring height. The magic scoring trajectory corresponds to a launch speed of approximately 42 ft/sec at an initial trajectory angle of approximately 36 degrees relative to the horizontal for the Team 20 Eclipse Robot, This is a function on the initial height of the ball as it sits in the robot.

There are several variables that determine the ball trajectory. Some are fixed at the fabrication stage, but others can be configured in the field for fine tuning the catapult trajectory. The design elements that are fixed include the following:

1) The spring rate of the springs
2) The maximum dynamic moment capability of the arm, pivot and spring support points. (The strength of the arm)
3) The lateral distance between the arms. Moving the arms further apart by changing the shims along the pivot axis can change the radial location of the ball.

Catapult parameters that can be varied to tune the trajectory (in the pit) in the order of their significance and the parameters that they effect are summarized in the table below:

	Primary Influence	Secondary Influence
Spring preload cam setting	Ball speed	Release angle (angular acceleration influences release angle)
Winch pull back angle	Ball speed	
Radial location of the back arms	Ball speed	Ball moment of inertia, Catapult moment of inertia
Height of the back arms	Ball initial trajectory angle	Catapult moment of inertia

	(release angle)	
Catapult moment of inertia	Ball speed - adjust with weights bolted to arm	

The spring moment (strength) has an large effect on the ball trajectory. The spring moment is mostly set by the selection of springs. The initial moment provided by the springs is adjusted in the field by means of course and fine adjustments.

Course adjustments include the number of springs engaged, and the preload on the springs as set by the preload "cam". It is envisioned that all launch scenarios will use all springs. Finer adjustment on the preload can be set by shimming the turn off switch that determines the stopping point of the winch when it winds up.

Another major field adjustable catapult parameter that determines the ball trajectory is the radial location and height location of the back arms. In practice the ball arms can even be angles. The back arms determine the speed of the ball at launch, the launch angle and the back spin on the ball. Exactly where the ball loses contact with the arm is difficult to precisely determine as the ball tends to roll off the back, but it is near the unloaded spring position which corresponds to a vertical launch angle of 30 degrees. Most likely it is slightly before the unloaded spring angle. The ball trajectory analysis assumes the ball has an initial launch angle of 36 degrees. Because the back arms are the furthest structure from the pivot, they are the component that contributes most to the moment area of inertia. The larger the inertia, the slower the launch speed.

The moment of inertia of the system has a large impact on the launch speed. The more inertia on the arm, the less its acceleration for a given spring preload. For the current system the breakdown of the inertia is approximately as follows:

payload (ball) moment area of inertia	744lbm*in^2
catapult moment area of inertia	402lbm*in^2 (including the effect of the winch)
System	1146lbm*in^2

Design Calculations

Calculations were performed to determine the predicted ball trajectory. There are three major areas of analysis that are needed to predict the ball trajectory:

1) Torsion spring rate and stress calculations

2) "Catapult as an spring, inertia system"

3) Mechanical stress in the arm

4) Ball kinematics including the effects of air friction, spin, and initial ball position and flight angle on the actual ball trajectory.

Torsion spring calculations and sizing

The relationship between moment and angular displacement in units of in*lbf / radian developed by a single spring is given by the following equation:

$$k_{spring}(d, D, nturn) := \frac{d^4 \cdot E_{spring}}{10.8 \cdot D \cdot nturn} \cdot \frac{1}{2 \cdot \pi}$$

where k is the spring rate

d is the wire diameter

E is the modulus

D is the outside diameter of the spring coil

nturn is the number of turns of the coil.

Within a springs linear range, the moment or torque developed by the spring is given by the following equation:

$$T_{pull} := k_s \cdot \theta_s$$

where T_{pull} is the moment (in*lbf) developed by the spring,

k_s is the spring rate of the system of springs

θ_s is the stretch of the spring (in*lbf / radian)

The approximate pull force exerted by the winch for the different shots are as follows:

sweet spot shooter	112lbf
"hail Mary" pass	144lbf

It is desirable to operate a spring in its linear region and such that the stresses in the spring are within the spring materials endurance limit. The parameters that extend the springs linear range (D, coil diameter and n turns) tend to reduce the magnitude of the spring rate. That is why the springs on the catapult are so large. Smaller springs could develop the required force, however, they would have to be stretched beyond their linear range and be subject to low cycle fatigue failure and a decrease in spring rate with usage.

Four springs are used in the catapult. The spring rate of four parallel springs is 4 times the spring rate of the individual springs. Springs in parallel "add". Springs in series add as their reciprocals. The multiple turns on a given torsion spring is an example of springs in series. The more "spring coils" in series, the lower the spring constant, but the greater range of linearity.

<u>Equations of motion for catapult as spring inertia system</u>

Up until the point at which the ball leaves contact with the arm, the mechanical system can be modeled as a "spring mass system". There is a negligible amount of damping action on the ball during its acceleration, so for all practical purposes the system is undamped. The inertia of the ball, the arm and the reflected inertia of the winch are all accounted for in the moment of inertia, Jpolar.

The governing equations of motion are as follows:

$$\theta(t) := (\theta_s)\cdot\cos\left(\sqrt{\frac{k_s}{J_{polar}}}\cdot t\right)$$ angular position as a fuction of time (radian)

$$\theta_{dot}(t) := \theta_s\cdot\sqrt{\frac{k_s}{J_{polar}}}\cdot\sin\left(\sqrt{\frac{k_s}{J_{polar}}}\cdot t\right)$$ angular speed as a functio of time (radian/sec)

$$\theta_{ddot}(t) := \theta_s\cdot\frac{k_s}{J_{polar}}\cdot\cos\left(\sqrt{\frac{k_s}{J_{polar}}}\cdot t\right)$$ angular acceleration as a function of time (rad/s^2)

$$v(t) := r_{caput}\cdot\theta_{dot}(t)$$ linear speed of the ball (ft/sec)

$$t_f := \sqrt{\frac{J_{polar}}{k_s}}\cdot\text{acos}\left(\frac{\theta_{release}}{\theta_s}\right)$$ time at which the ball is no longer in contact with the catapult

where the independent variables are defined as follows:
ks is the total spring constant of all the springs
Jpolar is the total polar moment of inertia of the ball, arm and reflected catapult
t is time,

The purpose of the catapult is to accelerate the ball to the desired initial speed and angle (velocity vector). The exact release angle is somewhat difficult to predict as depends on the arm height and the way the ball "rolls" off of the arm (while be in bumped around by "defending robots") , but it can be approximated. The catapult is designed so that the springs are unloaded near the anticipated release angle of approximately 36 degrees relative to the horizontal for a shot on the high goal.

It can be seen by the equations above that the initial ball velocity can be increase by increase the spring stretch angle, the spring rate and the radial distance of the ball from the pivot. The effect of the inertia is to reduce the launch speed. It is the inertia of the launch arm that ultimately limits the range of the catapult.

Mechanical loads on the arms

The largest loads in the catapult members are caused by the springs as they accelerate the ball. The forces acting on the main arms are highest at time 0 (maximum spring stretch). The forces acting of the "extension arms" is highest at the launch angle when the speed is the highest.

The spring provides the torque to accelerate the launch arm. The velocity profile, however, is limited by the moment of inertia of the arm and ball. It is desirable to have an arm with as low an inertia as possible so that the maximum speed can be achieved. The arm, however, must have sufficient mass to maintain rigidity and not break.

The maximum stress in the main arms occur where the bending moment is the largest. The bending moment is the highest where the springs contact with the arm. In general, the stress in a beam under static bending load is proportional to the applied moment, and the moment area of inertial of the arm cross section. A catapult is a dynamic (moving) system, however, so there are inertial effects which will cause the stress to be slightly higher than in a static system. A dynamic load factor is used to accommodate for inertial effects.

As is evident in Figure 1, the arm is "beefed up" where the dynamic moment is the highest.

There is also a significant centrifugal force on the extension arms that keep the ball along the arc path until it "rolls" off.

The ball follows a circular path as long as it is engaged with the catapult. The velocity is the angular speed times the radius of action on the ball. To keep the ball on a circular trajectory requires the arms to exert a considerable force on the ball. This force is on the order of 100 pounds. The equation for this forces is as follows:

$$F_{cball}(t) := \theta_{dot}(t)^2 \cdot r_{caput} \cdot m_{ball}$$

Ball Kinematics

Once the launch velocity (speed and position) has been determined, the trajectory of the ball can be calculated accounting for air friction and ball spin with reasonable assumptions. Air friction has a very significant effect on the trajectory. The spin is assumed to be 1 rotation per second, and has a lesser effect on the trajectory. The spin is in a direction so as to create lift and elongate the trajectory.

Air friction is accounted for assuming a drag coefficient of 0.3. The air friction force acting on the ball is given by the formula below:

$$F_{drag}(v_{ball}) := \frac{C_d \cdot \left(\frac{\pi \cdot D_{ball}^{2}}{4} \cdot \rho_{air} \cdot v_{ball}^{2} \right)}{2}$$

where

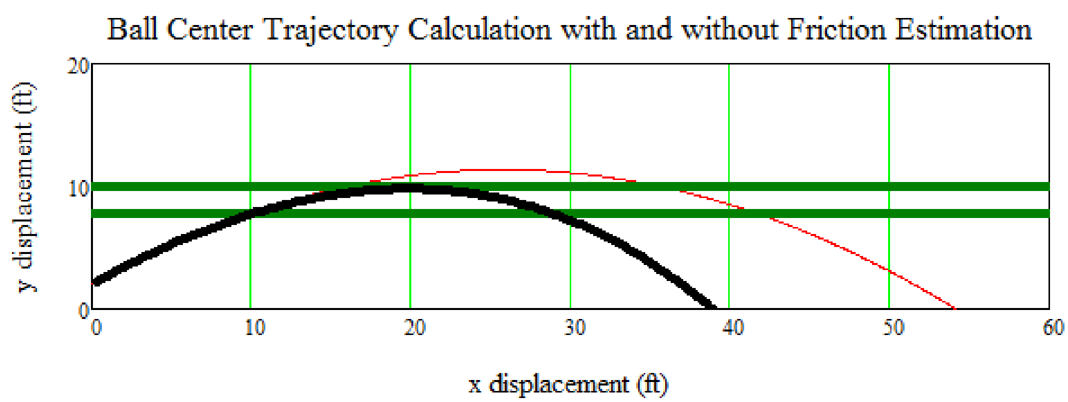
F_{drag} is the drag force caused by air friction
v_{ball} is the velocity of the ball
C_d is the drag coefficient
D_{ball} is the diameter of the ball
v_{ball} is the velocity of the ball

$\rho_{air} = 0.075 \frac{lbm}{ft^3}$ density of air

At launch the air friction force acting on the ball is approximately 1.6lbf.

The ball trajectory for a sweet spot shot (spring preload of approximately 70 degrees) with an initial velocity of 42 ft/sec at 36 degrees relative to the horizontal with and without the effects of spin and friction is shown in Figure 2 as the black trajectory. Note that a cross field diagonal shot has a longer trajectory than a shot taken perpendicular to the goals so the sweet spot shooter needs a range more than half of the field length.

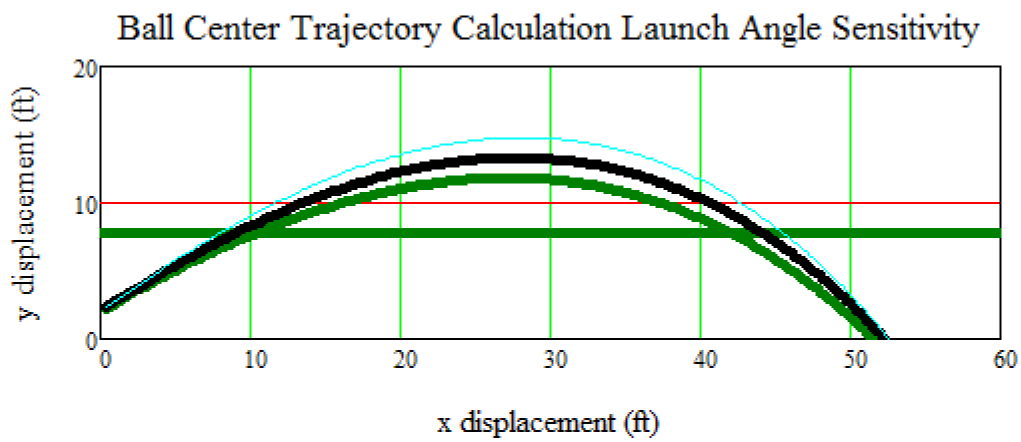
Figure 2: Ball trajectory for sweet spot shooter Configuration



The red trace shows the ball trajectory without the effects of friction and ball spin. The green lines are the heights that that correspond with a score in the goal. The goal of the sweet spot shooting mode is to score from the widest range possible.

For the "Hail Mary" system configuration where the spring is loaded preloaded by 90 degrees the ball trajectory for launch angles of 33, 36 and 39 degrees is predicted the be as shown in the Figure 3:

Figure 3. Ball trajectory for "Hail Mary" passer configuration



These trajectories ignore friction with the "catcher wings" which is often used to settle the ball in competition. Regardless, there is sufficient range in the design to increase the preload a little to overcome this frictional effect.

Conclusion

The Team 20 Eclipse robot utilized torsion springs in a catapult designed to be a sweet spot shooter. It can be reconfigured as a Hail Mary passer, or a close range shooter as well by adjustment of the spring preload.