2/17/2016 Ether

This paper shows the derivation of the formulas for computing the launch angle and the equations of the two parabolic (no air drag) trajectories, given the launch speed and the coordinates (d,h) of a point on the trajectories.

Equation for parabola with origin at launch point:

$$y = a * x^2 + b * x$$

Take the derivative:

$$\frac{dy}{dx} = 2 * a * x + b$$

when x=0:

$$\frac{dy}{dx} = b$$

and

$$\frac{dy}{dx} = \tan(\theta)$$

so

$$b = \tan(\theta)$$

so

$$\cos(\theta) = \frac{1}{\sqrt{b^2 + 1}}$$

and

$$\sin(\theta) = \frac{b}{\sqrt{b^2 + 1}}$$

Let the launch speed be Vo, then:

$$Vox = Vo * \cos(\theta)$$

$$= Vo * \frac{1}{\sqrt{b^2 + 1}}$$

and

$$Voy = Vo * \sin(\theta)$$

$$= Vo * \frac{b}{\sqrt{b^2 + 1}}$$

The time t to get to horizontal distance d is:

$$t = \frac{d}{Vox}$$

$$= d * \frac{\sqrt{b^2 + 1}}{Vo}$$

The height must be h at that same time:

$$Voy * t - \frac{g}{2} * t^2 = h$$

substituting for Voy and t:

$$b * d - \frac{(b^2 + 1) * d^2 * g}{2 * Vo^2} = h$$

expanding and re-arranging:

$$d^2 * \frac{g}{2 * Vo^2} * b^2 - d * b + \left(d^2 * \frac{g}{2 * Vo^2} + h\right) = 0$$

Let

$$K = \frac{g}{2 * Vo^2}$$

Then

$$d^2 * K * b^2 - d * b + (d^2 * K + h) = 0$$

Compute the constant K one time (since Vo is constant):

$$K = \frac{g}{2 * Vo^2}$$

Now whenever you need the angle for a new distance d or height h, compute in the following order:

$$A = d^2 * K$$

$$B = -d$$

$$C = A + h$$

$$b_1 = \frac{-B - \sqrt{B^2 - 4 * A * C}}{2 * A}$$

$$\theta_1 = \mathrm{atan}(b_1) * \frac{180}{\pi}$$

$$b_2 = \frac{-B + \sqrt{B^2 - 4 * A * C}}{2 * A}$$

$$\theta_2 = \mathrm{atan}(b_2) * \frac{180}{\pi}$$

To find the equation of the parabola:

$$y=a*x^2+b*x$$

The point (d,h) is on the parabola, so:

$$h=a*d^2+b*d$$

Solve for a to get:

$$a=\frac{h-b*d}{d^2}$$

Plug that back into the parabola equation:

$$y=\frac{(h-b*d)*x^2}{d^2}+b*x$$

Then substitute the numerical values of h, d, and b1 or b2 into the above equation