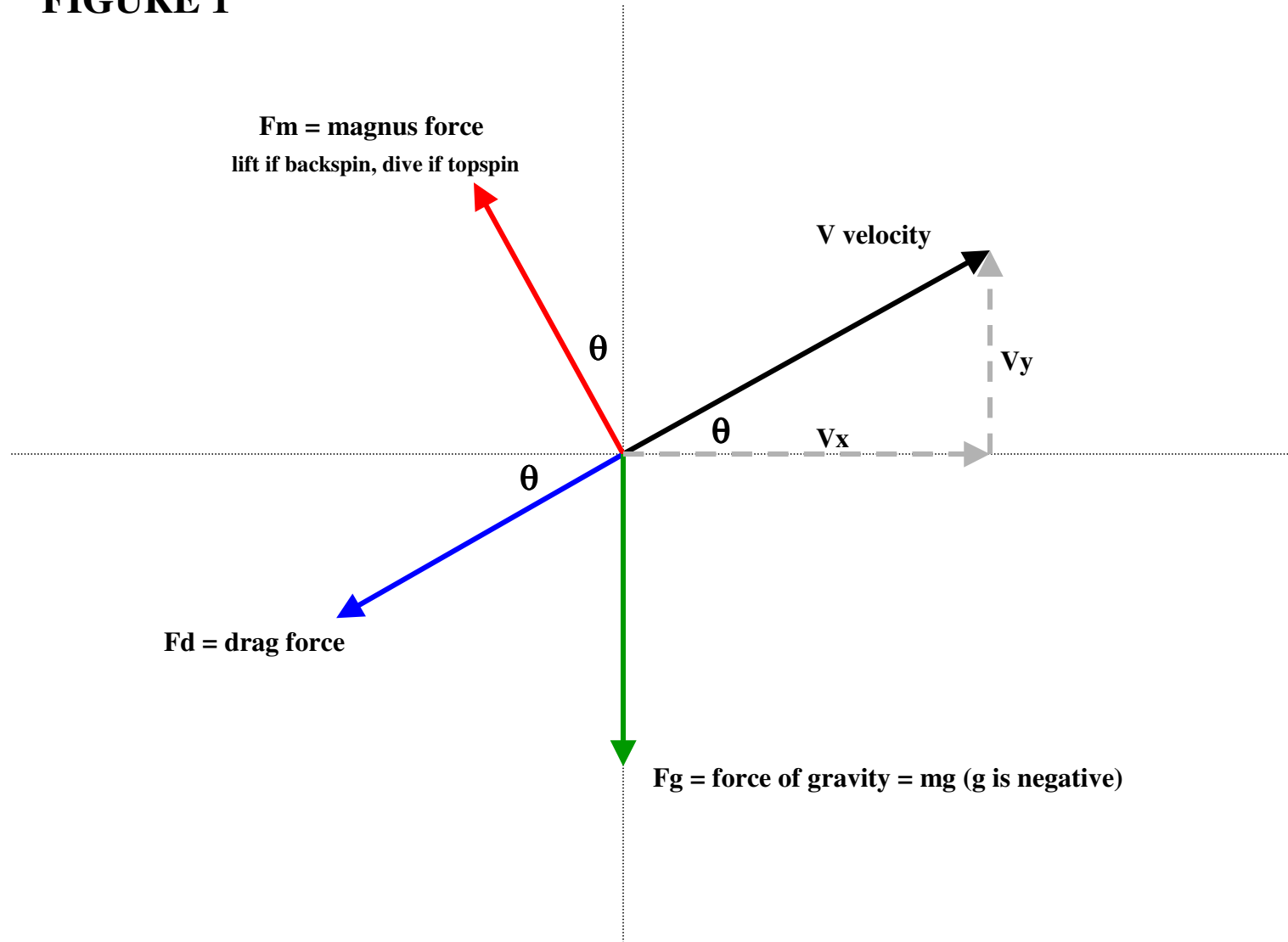


Rebound Rumble
Ballistic Trajectory
with air friction drag
and
backspin/topspin lift/dive (Magnus effect)

Free-body force diagram, derivation of differential equations, and example C pseudo-code for trapezoidal numerical integration

FIGURE 1



Refer to FIGURE 1 on previous page

$$V = \text{sqrt}(V_x^2 + V_y^2)$$

$$\sin(\theta) = V_y / V$$

$$\cos(\theta) = V_x / V$$

$$F_d = (1/2) \cdot \rho \cdot A \cdot C_D \cdot V^2 \quad (\text{see Note 1 on last page})$$

$$\text{Let } \mu_d = (1/2) \cdot \rho \cdot A \cdot C_D \quad \dots \text{ then } F_d = \mu_d \cdot V^2$$

$$F_m = (1/2) \cdot \rho \cdot A \cdot C_L \cdot V^2 \quad (\text{see Note 2 on last page})$$

$$\text{Let } \mu_m = (1/2) \cdot \rho \cdot A \cdot C_L \quad \dots \text{ then } F_m = \mu_m \cdot V^2$$

continued on next slide...

...continued from previous slide:

$$F_x = -F_m \sin(\theta) - F_d \cos(\theta)$$

$$F_y = F_g + F_m \cos(\theta) - F_d \sin(\theta)$$

$$a_x = F_x / M = -(V/M)(\mu_d V_x + \mu_m V_y)$$

$$a_y = F_y / M = (V/M)(\mu_m V_x - \mu_d V_y) + g$$

C pseudo-code for trapezoidal numerical integration of Differential Equations

```
// first initialize all variables, then:

while (y>=0) {

Vxn = Vx + dt*ax; // assume constant acceleration during dt
Vyn = Vy + dt*ay;

x = x + dt*(Vx+Vxn)/2; // trapezoidal integration of position
y = y + dt*(Vy+Vyn)/2;

Vx = Vxn; // update Vx&Vy for next iteration
Vy = Vyn;

V = sqrt(Vx^2 + Vy^2); // temporary variable

ax = -(V/M)*(ud*Vx+um*Vy); // update ax&ay for next iteration:
ay = g+(V/M)*(um*Vx-ud*Vy);

t = t + dt;

// output t, x, and y here
}
```

The last two equations on Slide 4 are the differential equations of motion for the projectile, re-written here as time derivatives of \mathbf{x} and \mathbf{y}

$$\ddot{x} = -\frac{\sqrt{\dot{x}^2 + \dot{y}^2}}{M} (\mu_d \dot{x} + \mu_m \dot{y})$$

$$\ddot{y} = \frac{\sqrt{\dot{x}^2 + \dot{y}^2}}{M} (\mu_m \dot{x} - \mu_d \dot{y}) + g$$

Notes:

1) http://en.wikipedia.org/wiki/Drag_equation

2) http://en.wikipedia.org/wiki/Magnus_effect#An_example_of_spin_ball_in_the_air