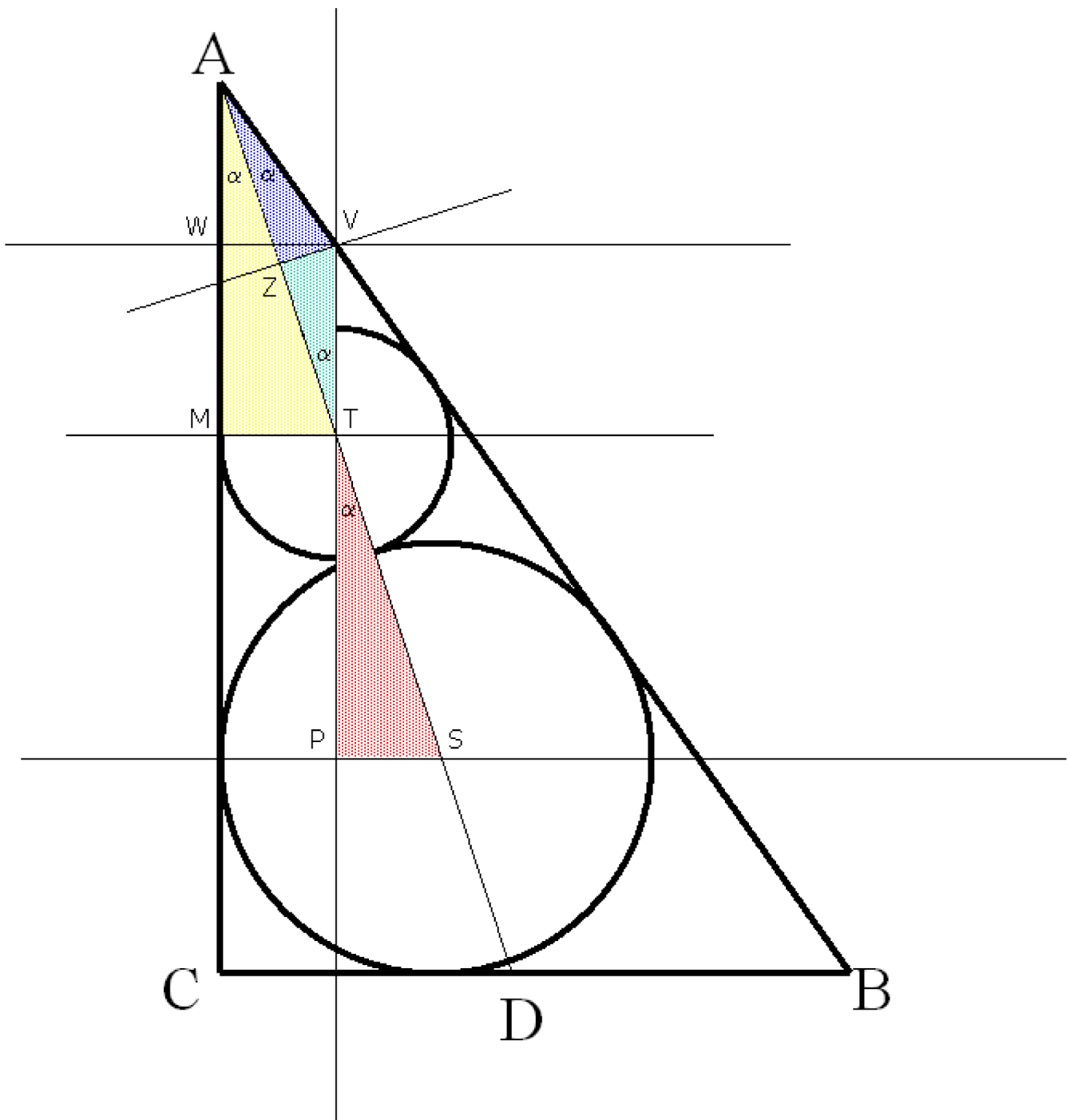


Math Quiz 8 - Geometric Solution

Figure 1:

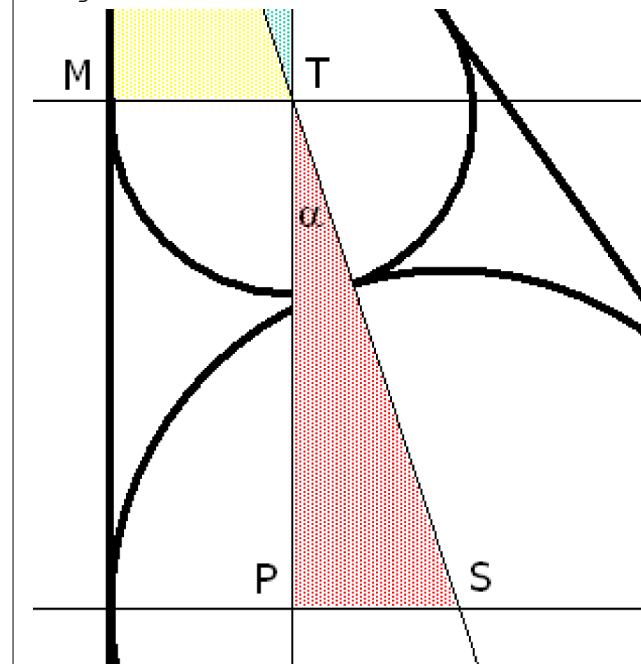


Refer to Figure 1 above.  
 $AD$  is the bisector of angle  $ABC$ .  
 $WV$ ,  $MT$ , and  $PS$  are parallel to  $CB$ .  
 $VP$  is parallel to  $AC$ .  
 $VZ$  is perpendicular to  $AT$ .  
 $T$  and  $S$  are the centers of their respective circles.

Without loss of generality, let  $r=1$

```
(%i1) r:1$
      R:K*r$
      MT:r$
      WV:r$
```

Figure 2:



Hypotenuse TS of red triangle is  $R+r$ :

```
(%i5) TS: facsum(R+r);
(%o5) K+1
```

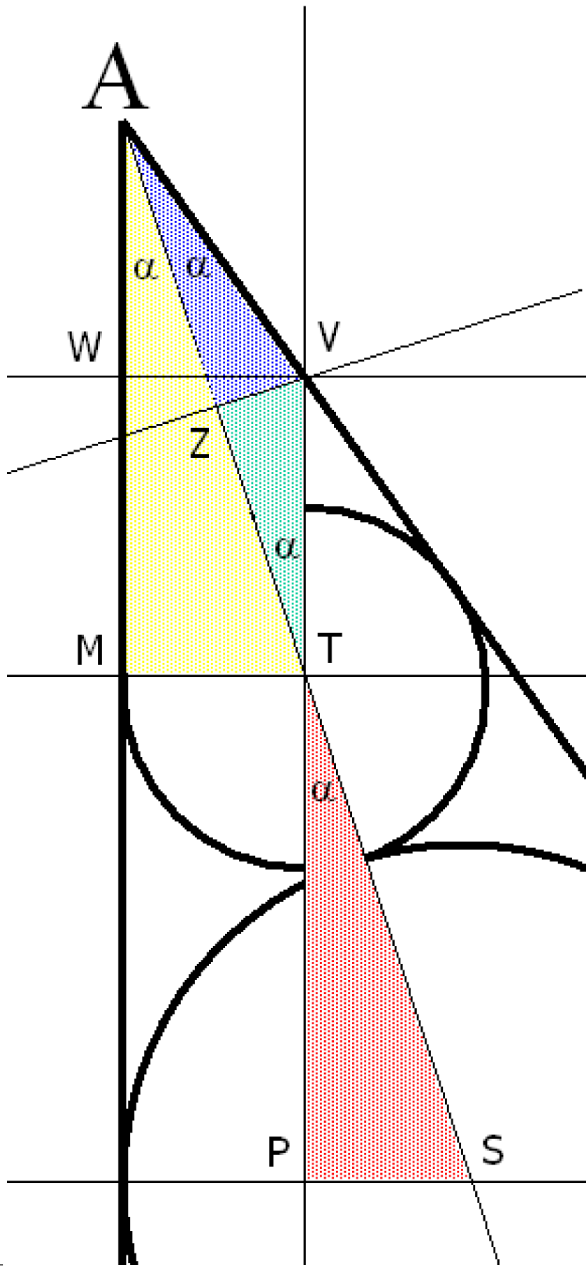
Base PS of red triangle is  $R-r$ :

```
(%i6) PS: facsum(R-r);
(%o6) K-1
```

Use Pythagoras to get altitude TP of red triangle:

```
(%i7) TP: ratsimp( sqrt( TS^2 - PS^2 ) );
(%o7) 2*sqrt(K)
```

Figure 3:

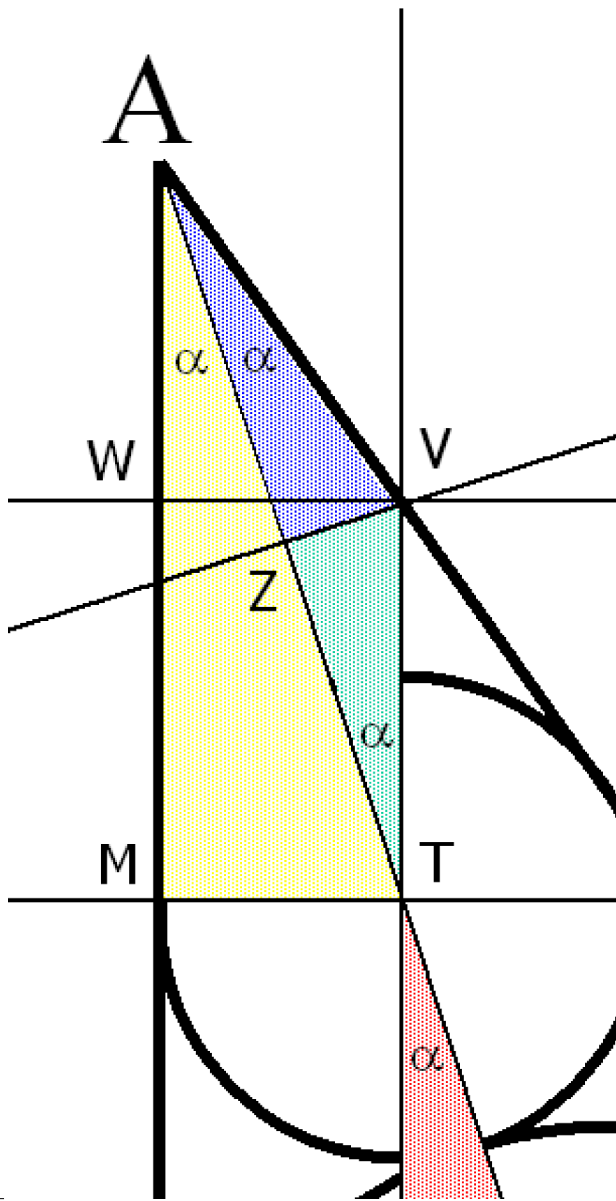


Use similarity of yellow and red triangles to find hypoteneuse AT of yellow triangle:

(%i8)  $AT: MT*(TS/PS);$

(%o8)  $\frac{K+1}{K-1}$

Figure 4:

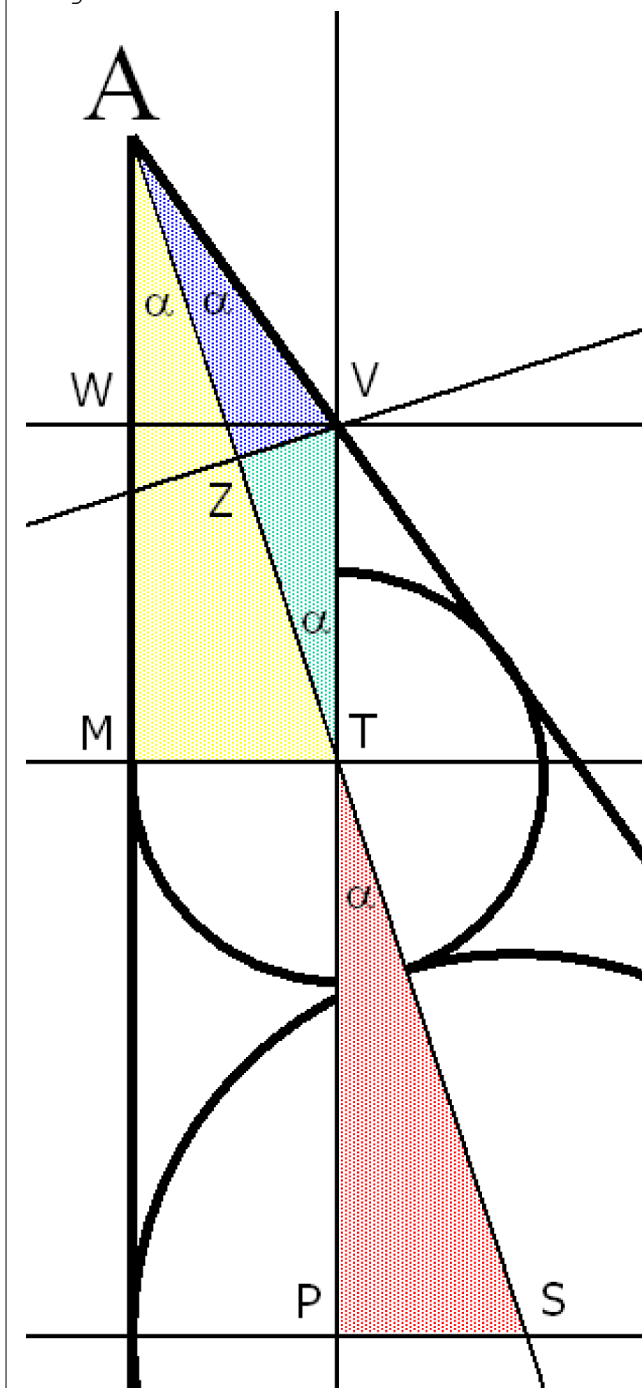


Green and blue triangles are congruent,  
so AZ is half of AT:

$$AZ = AT/2;$$

$$\frac{K+1}{2(K-1)}$$

Figure 5:

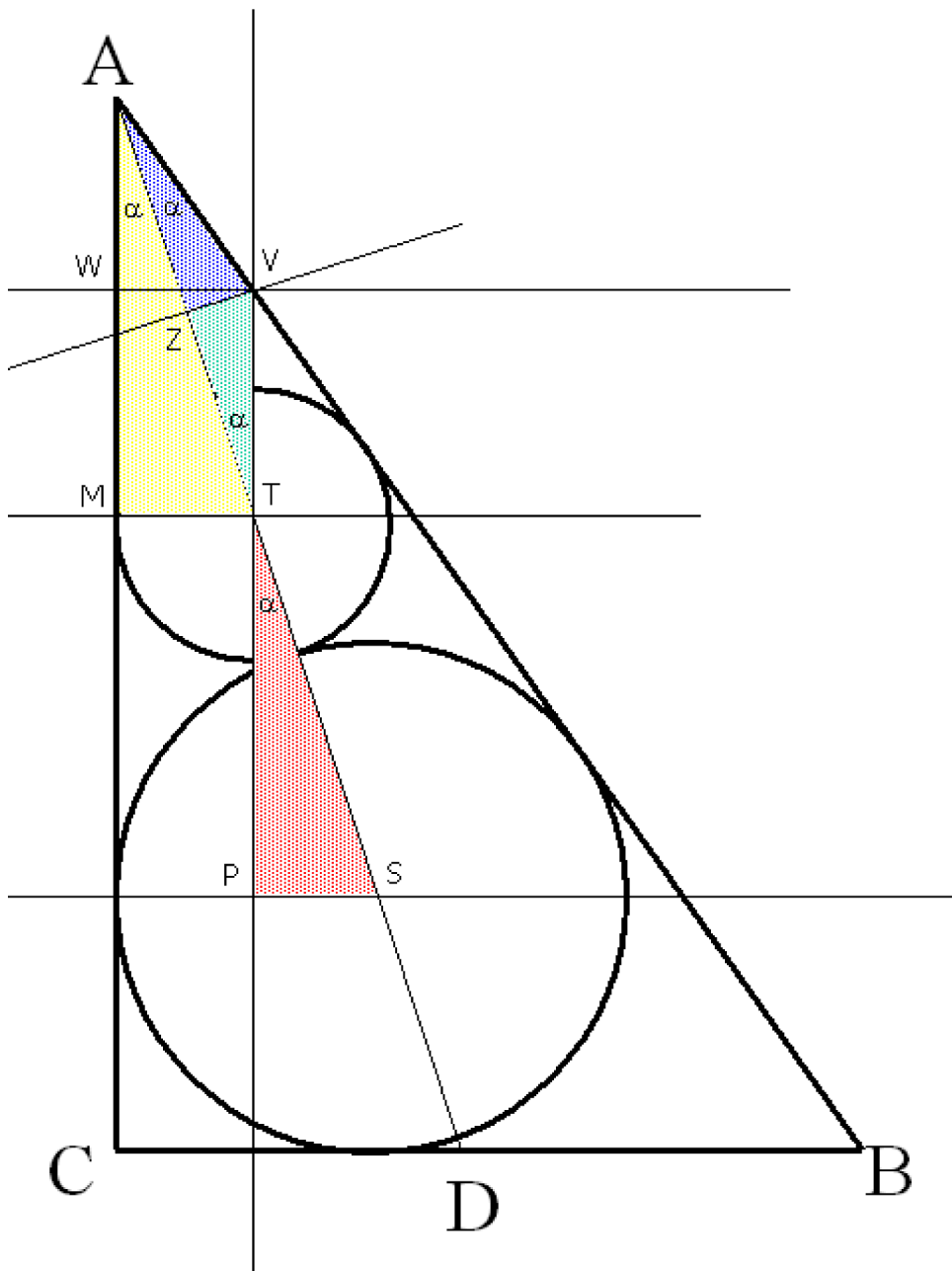


Use similarity of blue and red triangles to find hypotenuse AV of blue triangle:

$$\text{AV} = \text{AZ} \cdot \left(\frac{\text{TS}}{\text{TP}}\right);$$

$$\frac{(K+1)^2}{4(K-1)\sqrt{K}}$$

Figure 6:



Triangles  $AWV$  and  $ACB$  are similar,  
so  $AB/CB$  is equal to  $AV/WV$

$$AB/CB = \frac{(K+1)^2}{4(K-1)\sqrt{K}}$$