Power consumption limits and total motor power in FRC drives

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1 Introduction

Which motors, and how many of them, should be put on an robot drive? This question is a topic of much discussion and little agreement in the FRC community. In recent years, in particular, the issue has come into focus, as the number of teams using "high-power" drives has skyrocketed due to the increased availability and variety of reliable commercial off-the-shelf gearboxes, and the liberal motor budget and emphasis on mobility of recent FRC games.

As the ubiquity of these drives has increased, so too has questioning of their actual merit in terms of practical robot performance. While much theoretical discussion has taken place on Chief Delphi (especially around the popular "6-CIM" drive) and several spreadsheets and software calculators for simulating the performance of drives in various configurations are available, much of the analysis remains focused on the motor and battery dynamics alone.

As this whitepaper will demonstrate, this overlooks some likely nontrivial effects stemming from the interactions between the functionality of the H-bridges used as motor controllers in FRC, the limitations on maximum possible current draw from the FRC battery due the roboRIO's brownout protection, and the total available motor power on the robot. In particular, there appears to be a seldom-discussed potential benefit to increasing the amount of total motor power on the robot even in a condition in which the robot's performance is "power-limited."

2 The FRC battery, current limits, and H-bridge functionality: Why FRC drives are power-limited (at low speeds)

It is a well-known fact that a typical FRC drive is capable of drawing more current than the FRC battery can effectively provide.

A typical FRC drive consists of four CIM motors, each with a stall current of $\sim$130 A. Given a perfect voltage source, then, stalling all four motors would draw

$\sim$520 A. However, the FRC battery is not a perfect voltage source. A typical FRC battery in good condition will have an internal resistance of $\sim$0.02 Ω. Thus, our theoretical current draw would result in a voltage drop of $\sim$10 V. As a charged battery has a no-load voltage of $\sim$13 V, and the FRC control system enters brownout protection at $\sim$7 V, clearly such a current draw is not possible. A good rule of thumb (that can be verified with some simple algebra) is that one cannot draw more than $\sim$250 A from the FRC battery without risking control system brownout.

This much is well-established, and should be familiar to most readers. Since triggering the roboRIO's brownout protection is almost always undesirable (in that it causes unpredictable shutting-off of the motors to reduce current draw), many teams have taken to implementing some form of "current-limiting" on their drives, scaling back the motor output to keep current draws within acceptable ranges in a controlled manner. This has become particularly widespread due to the inclusion of built-in current-limiting functionality on the popular Talon SRX motor controller, and to a lesser extent to the current draw monitoring supported by the current version of the power distribution panel.

However, the throttling of motor outputs to satisfy a current limit introduces some seldom-discussed subtleties to the analysis of motor behavior. In particular, most analysis of motor behavior in FRC is done under the implicit assumption that motors are being run at full-throttle.

FRC motor controllers control the voltage supplied to motors through a "duty cycle." In short, this is accomplished by rapidly switching the circuit to the motor open and closed, with the average voltage seen by the motors controlled, roughly, by the fraction of the time that motor is connected to the battery. Thus, a "50% duty cycle" would indicate that the "on" and "off" phases of the cycle are equal in length, and the motor is receiving an average of half of the bus voltage.

For most purposes, it suffices to conceptualize this as simply supplying the corresponding fraction of bus voltage to the motor. However, when *current* is considered, the situation grows quite a bit more complicated. In brief, during the "off" phase of a motor controller's duty-cycle, the motor leads are shorted together, allowing current to continue flowing through the motor.[1] As the transient decay-time for the current is long compared to the switching frequency, the result is that the current *through the motor* stays more-or-less constant throughout the cycle. However, current is only drawn *from the battery* during the "on" phase of the duty cycle, and so the *average* current drawn from the battery is lower than the average current through the motor, in a manner roughly proportional to the duty cycle (i.e., at a 70% duty cycle, the average current drawn from the battery will be $\sim$ 70% of the average current through the motor.

It may seem that this is violating some sort of conservation law. However,

[1] To see why this is done, consider what would happen if the circuit were simply left open - in this case, the current would be forced to fall to 0 almost immediately during the "off" phase of the duty cycle. Since current is proportional to torque in a DC motor, this would severely limit the torque output of the motor - additionally, energy lost as the current falls to 0 would have to be dissipated in the motor controller, which is, for obvious reasons, highly-undesirable.

recall that *power* is equal to the product of voltage and current - and while the current supplied to the motor may be higher than the current drawn from the battery, we can see that the voltage supplied is proportionally lower, and so the total power supplied is, indeed, conserved.[2]

Now, recall our original cause for current-limiting - we wish to limit the current drawn *from the battery.* Indeed, most software current limits (including the built-in functionality in the Talon SRX) are implemented to limit this, and not the current supplied to the motors. Thus, in the spirit of the previous paragraph, since current is not actually conserved across the motor controller it is actually more appropriate to think of these limits as *power* limits rather than current limits. We can thus take our earlier heuristic 250 A limit and, multiplying by the approximate battery voltage corresponding to such a voltage drop, re-phrase it in terms of power: A good rule of thumb is that one cannot draw more than ~1750 W from the FRC battery without risking control system brownout.

As a final note, it is reasonably clear that this limitation is only really important at low speeds - at high speeds, almost no configuration of drive motors will consume enough power to come anywhere near this limit.

3 Total motor power, heat dissipation, and an unexpected quirk

So, what has this all got to do with total motor power on a robot?

We established in the previous section that (at low speeds) FRC drives using current-limiting to avoid brownout are effectively constrained by a power limit. To determine how a robot drive behaves under this power limit, we will have to do some work with the motor equations. A naive approach would be to find the equation describing the amount of power consumed by a motor, use this to figure out the total power consumed by the drive, set this equal to the power limit, and see what happens. Let's do that:

The power consumed by a motor can be partitioned into two parts - mechanical power output to the shaft, and heat dissipated in the motor coils. Call these P_{mech} and P_{heat}. As it happens[3]

$$P_{mech} = V_{emf} \cdot I_{motor} \tag{1}$$

and

$$P_{heat} = R_{motor} \cdot I_{motor}^2 \tag{2}$$

Where V_{emf} is the back-EMF of the motor (this is proportional to the rotor speed), I_{motor} is the current through the motor, and R_{motor} is the resistance of

[2]If it were not, we would indeed have cause for suspicion - conservation of energy is very much a certainty!

[3]Exercise for the reader: verify that these equations are true, particularly the first one

the motor. Combining these, we obtain[4]

$$P_{total} = P_{mech} + P_{heat} = V_{emf} \cdot I_{motor} + R_{motor} \cdot I^2_{motor} \tag{3}$$

We now have enough information, the specifications of a robot's drive motors, and a fixed speed at which it is traveling, to solve for the current *in the motors* under the constraint of a power limit. The general solution, however, is rather ugly[5] and does not admit any particularly-simple descriptive results, and so for the purposes of this paper we'll limit ourselves to the case of a robot at stall.[6]

For a robot at stall, P_{mech} is zero - all the power consumed is dissipated as heat, and thus $P_{total} = R_{motor} \cdot I^2_{motor}$. This allows us to very quickly arrive at some interesting conclusions - for example, let us investigate the effect of an increase in motor power by comparing the behavior of a single motor to two identical motors in parallel, at stall, under the same power expenditure. Equating the power expenditure and applying our earlier equations, we see

$$R_{motor} \cdot I^2_{single} = 2 \cdot R_{motor} \cdot I^2_{parallel} \tag{4}$$

where I_{single} is the current through the single motor, while $I_{parallel}$ is the current through *each* parallel motor. Simplifying this yields

$$I_{single} = \sqrt{2} \cdot I_{parallel} \tag{5}$$

Since torque output is proportional to current in a DC motor, we see from this that the single motor outputs only $\sqrt{2}$ times the torque of each of the parallel motors; as there are two of these, we can see that the parallel motors in sum actually output a factor of $\sqrt{2}$ more torque than the single motor! This can be easily seen to generalize: n motors stalled in parallel will output a factor of $\sqrt{n}$ more torque than a single motor, while consuming the same amount of power.[7]

This result is interesting, and somewhat counterintuitive - when not considering explicitly the case of a *power* limit, it would seem there is no possible benefit of adding more motor power to a FRC drive past the point where the

[4]If we are astute, we might further notice that with some clever factorization we can interpret the second equation in a manner similar to the first, by rewriting $R_{motor} \cdot I^2_{motor}$ as $V_{res} \cdot I_{motor}$, where V_{res} is the voltage drop across the motor due to resistance in the windings. This allows a somewhat more-elegant combined equation: $P_{total} = (V_{emf} + V_{res}) \cdot I_{motor}$. We can immediately see that this is equivalent to our earlier expression for total power, $P_{total} = V_{motor} \cdot I_{motor}$, verifying the initial claim that the total power can be partitioned in this way.

[5]It can be obtained from our combined equation through an application of the quadratic formula.

[6]Despite the lack of a neat formula, we can still get a sense about the behavior of the general case by extrapolating intuitively from the special case based on the form of the combined equation.

[7]A little bit more work will reveal that precisely the same result holds if, instead of considering identical motors in parallel, we consider a single "more-powerful" motor, if the "more-powerful" motor differs only by having a lower resistance and is identical in all other respects.

total stall current outstrips that which can be provided by the battery. However, a more careful analysis reveals that this is not the case - there is a clear benefit from the additional motor power. The question remaining, of course, is whether this benefit is of practical importance to a typical FRC robot.

4 Size of the effect in practice: calculations for some typical FRC drive configurations

Fortunately, we have the tools here to provide at least some insight into the aforementioned practical question. While empirical testing is obviously the ideal way to settle the question definitively, we can easily apply our earlier equations to calculate the torque-output-at-stall of some typical FRC motor configurations under the total power limit of 1750 W determined in the first section.[8][9] The values resulting from this computation are presented in the table below - to facilitate interpretation, total force tangent to the wheels of a drive geared for a free speed of $15\,\mathrm{ft\,s^{-1}}$ is reported rather than the net torque of the motors.

Drive Configuration	*Force Output at Stall* (lb)
4x CIM	138.8
6x CIM	170.0
6x mini-CIM	131.1
4x 775Pro	142.7
6x 775Pro	174.9
8x 775Pro	201.8

While we should be cautious in reading too much into these numbers - FRC robots are rarely at stall - the differences seen certainly *look* big enough to be significant.

5 Conclusions

So, what can we conclude from all this? As always, not as much as we'd like - but I think it is safe to say that there is a subtle, but potentially important, effect of adding more motor power to a drive that is power-limited.

While the benefit of increased efficiency when adding more motors is often mentioned in discussions of "high-power" drives and their merits, it is important to note that that effect is *not* the same as the effect described in this paper. Every calculation in this paper was done under a stall condition, and thus the efficiency is rigorously equal to zero for all the above calculations.

[8] To do this, we will simply set P_{total} for each motor equal to $\frac{1750\,\mathrm{W}}{n}$ where n is the total number of motors in the drive, and then solve for the current per motor, which likewise determines the torque output per motor and thus the total torque output of the drive.

[9] The 2x CIM and 4x mini-CIM configurations are not included here, not because they are not used, but rather because they are not actually capable of drawing 1750 watts from the battery, and thus are never "power-limited".

If we return our attention to our original power expenditure equations, we can intuit that the magnitude of this effect is greatest when in the stall condition that we investigated, and decreases as robot speed increases, since the P_{mech} term is merely linear in motor current, rather than quadratic. Moreover, we can also easily notice that this effect disappears when speed is high enough that the motors are no longer limited by the overall power limit imposed by the battery and the control system.[10] However, other benefits, such as the increased efficiency mentioned above, do apply at nonzero speeds.

More investigation - *especially* empirical investigation - is warranted to determine the ultimate practical impact of additional motor power on FRC drive performance. However, short of empirical investigation (which is, admittedly, difficult and resource-intensive), there is some immediate progress to be made on the theoretical front. Firstly, the calculations done at stall in this paper can easily be done at other, arbitrary fixed speeds. By running the calculation at an array of different speeds for each drive configuration - up to the speed at which the effect disappears - a more-complete picture could be generated of the (theoretical) performance differences.[11] Moreover, common FRC drive calculators and simulators could be updated to include a "power-limited" calculation at low speeds. Such a calculation may actually be simpler than the existing battery voltage sag and current draw calculations, as these do not need to be computed explicitly in this approach.

[10]Exercise to the reader: calculate the speed at which this occurs for each of the drives listed above.

[11]The results of such a computation could also be compared to the results of an "ordinary" computation that does not take the overall power limit into account, in order to isolate the size of this particular effect.