Ether 2/20/2014

There are two common ways to write the equation for a parabola in a Cartesian coordinate system whose Y axis is parallel to the parabola's axis:

1) $y = a*x^2 + b*x + c$

and

2) $y = a*(x-xp)^2 + yp$

In the first form, the y-intercept (where x=0) is simply $y=a*0^2+b*0+c = c$.

The slope at any point is given by $m = 2*a*x + b$.

At the apex m=0 so $0=2*a*x+b$, and the coordinates (xp,yp) at the apex are $xp=-b/(2*a)$, and $yp=c-b^2/(4*a)$

In the second form, the slope at any point is $m = 2*a*(x-xp)$.

At the apex m=0 so $0=2*a*x(x-xp)$ => x=xp at the apex.

When x=xp, $y=a*(xp-xp)^2+yp=yp$, so the coordinates of the apex are (xp,yp).

When x=0, $y=a*(xp)^2+yp$, so the y-intercept is $yi = yp+a*xp^2$

The trajectory of a projectile (gravity only; no air drag) in a uniform gravitational field is a parabola. With this year's game piece, speeds, and distances, air drag may be significant. With air drag modeled as proportional to speed squared, there is no closed-form solution, so numerical integration is used.

But it is still useful (and instructive) to learn about and use parabolic models; especially since the math is quite simple and everything can be solved analytically.

The equation for the parabolic trajectory of the game piece in a Cartesian coordinate system in the plane of the trajectory and whose origin (0,0) is at the position of the center of the game piece at the moment it becomes free of the launcher can be completely and uniquely specified[a] by any of the following sets of inputs:

1) launch speed and angle[b]

2) xy coordinates of the apex

3) xy coordinates of any point on the trajectory, and the slope at that point

3) xy coordinates of any two arbitrary points on the trajectory

4) y coordinate of the apex, and one arbitrary point on the trajectory

5) y coordinate of the apex, and the horizontal width W of the parabola a distance D below the apex

6) Launch angle and xy coordinates of any other point on the trajectory[b]

There are other combinations of inputs. But if you try to specify too many inputs you wind up with an overdetermined system of equations for which generally there is no solution. For example, you cannot specify both #4 above and the launch angle.

Once you've provided the inputs necessary to specify[a] the equation for the parabola, you can then use that equation to compute all the other properties of interest.

Notice that #5 above allows you to calculate the parabolic trajectory that produces a desired "scoring range". Set the y coordinate of the apex equal to bottom-h+goal-radius, set D equal to goal-2*radius, and set W equal to the desired scoring range[c].

Endnotes:

[a] the parabola is uniquely and completely specified by using the provided inputs to calculate values for a,b,c or a,xp,yp

[b] assuming "g" acceleration due to gravity is given

[c] bottom=6.8958; goal=3.0833; radius=1, h=height of (0,0) above the floor)