Adaptive Pure Pursuit

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Dawgma Robotics

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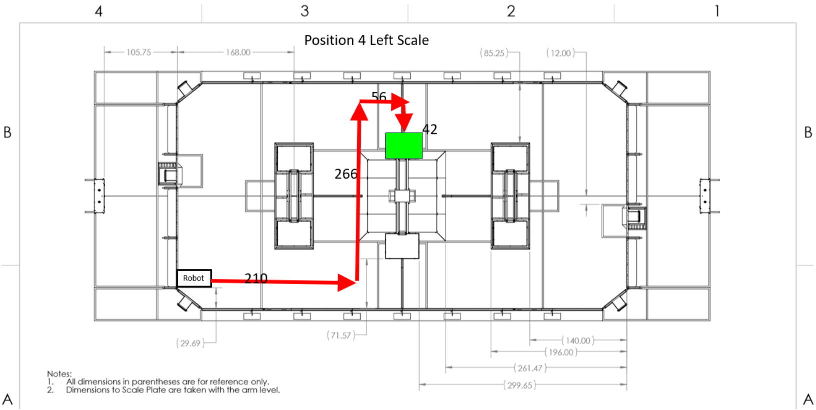
Introduction

- An overview of Pure Pursuit as used by team 1712 during the 2018 season
- Architectural and Mathematical overview. Staying away from code
- Please raise your hand to ask questions as you have them.

Brief History

- 16 Possible Paths
- Pure Pursuit Algorithm
- File-Encoded Routines

16 Possible Paths



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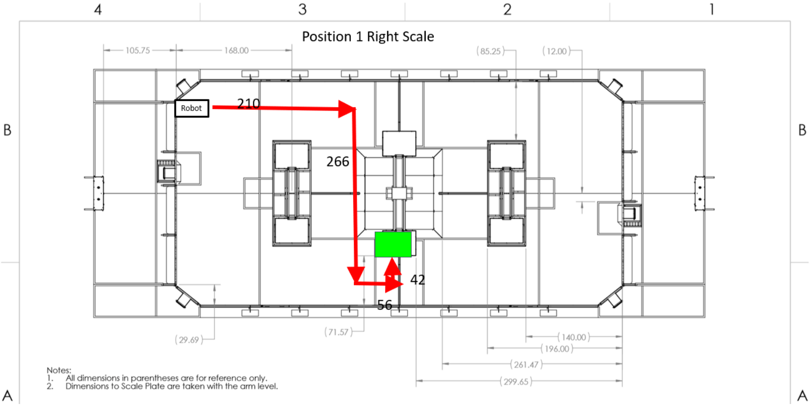

What is Pure Pursuit

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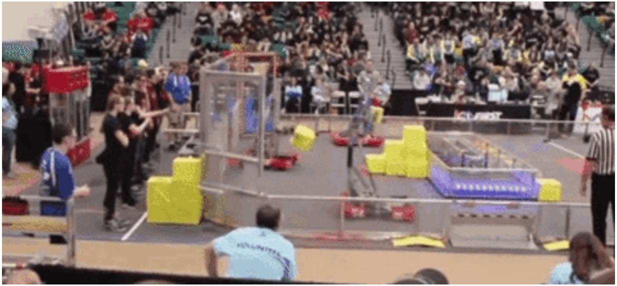
Analogy

Think of path generation as **drawing a virtual line**.
And think of path following as **walking along the virtual line**.

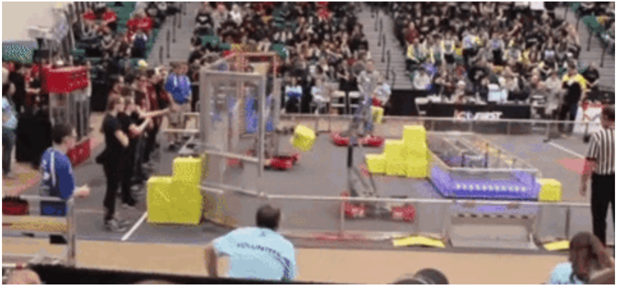
Odometry

- Use sensors to track the location of the robot
- Plot on a Cartesian Plain
- Pure Pursuit requires accuracy

Importance of Odometry



Importance of Odometry



- NavX failed
- Robot attempting to turn slightly left
- No input causes RoboRIO to believe that it is not turning at all
- Increasing control to attempt left turn

Sensors involved

- Rotary Encoders (one on each side of the drive train)
- NavX MXP for accurate angle

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- Rotary Encoders (one on each side of the drive train)
- NavX MXP for accurate angle
- Preset starting location
- Long term summation of changes to the position

Odometric Calculations

- D_l and D_r Distance traveled by the left and right wheels since previous iteration
- A angle robot is facing relative to the field.
- X_{prev} and Y_{prev} location from previous calculation
- X and Y location of the robot relative to starting position.

$$D = (D_l + D_r)/2 \tag{1}$$

$$X = X_{prev} + D * cos(A) \tag{2}$$

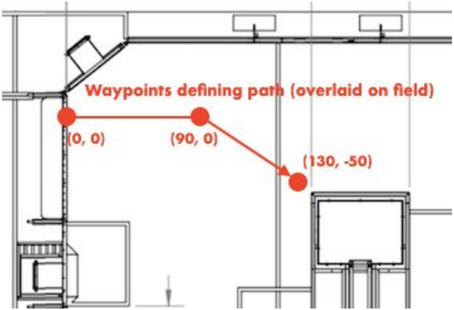
$$Y = Y_{prev} + D * sin(A) \tag{3}$$

Generating Paths

- Define start point, destination and way points
- Inject additional way points
- Smooth the path
- Curves and maximum speed along the path.

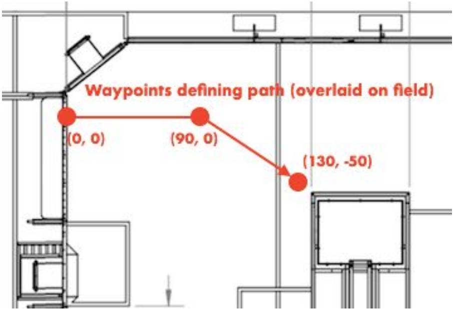
Define Way points

- Path Drawer Tool

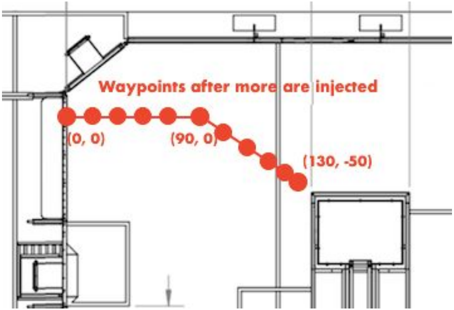


Define Way points

- Path Drawer Tool
- Start Point: one of four set positions.
- Way Points, to avoid obstacles.
- Destination.



Injecting Points



Injecting Points

- Path is a collection of way points
- Also a collection of line segments
- To inject points drop breadcrumbs at regular intervals

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```
interval := the distance between injected points;
segments := the lines between the way points;
newpoints := [ empty list of points ];
for each segment in segments:
    walker := segment.start;
    while (walker < segment.end):
        newpoints.append(walker);
        walker.advanceOnLine(segment, interval);
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```

- Dawgma used 6 inch sub-segments

Smoothing the Path

- Dawgma used the same algorithm as Team 2168
- Each point is a weighted combination of:
  - the original point
  - the midpoint of the previous and next points
- Repeats calculation of small increments
- Finishes when calculation results in sufficiently small changes (Tolerance)

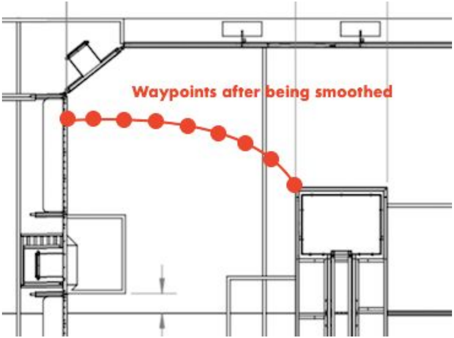
Smoothing the Path

```
let og, nc := original path, smoothed path (copy og);
let a, b := original weight, smoothing weight;
let t, c := tolerance, 0.0;
while(c >= t):
    c := 0;
    for each x, y in nc, og:
        let tmp := nc;
        nc +=
```

$$a(og - nc) + b(nc_{prev} + nc_{next} - 2(nc))$$

```
        c += absval(tmp - nc);
```

Smoothing the Path



Smoothing Alternatives

- Quintic Splines for smoothing way points

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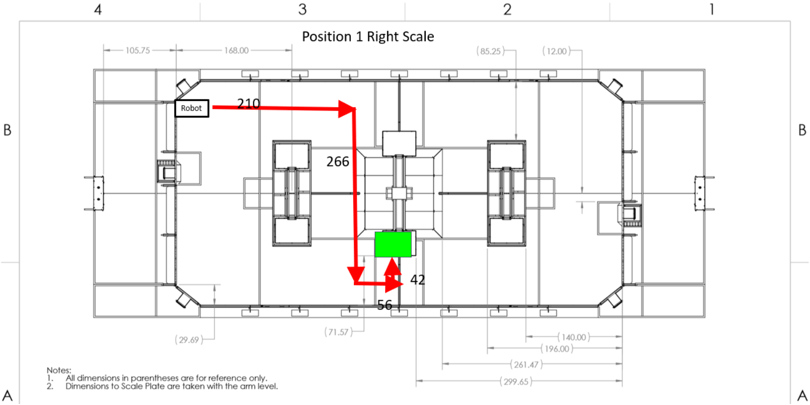
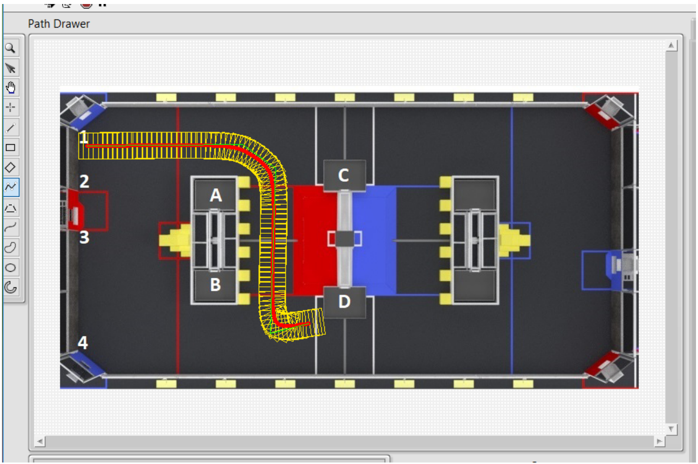
- Quintic Splines for smoothing way points
- Bezier Curves for directly generating a path
- Generate some points by hand

Visualizing Paths



Visualizing Paths



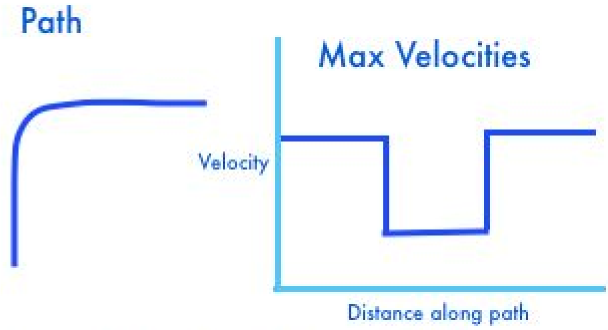
Curvatures and Velocities

- Story time.

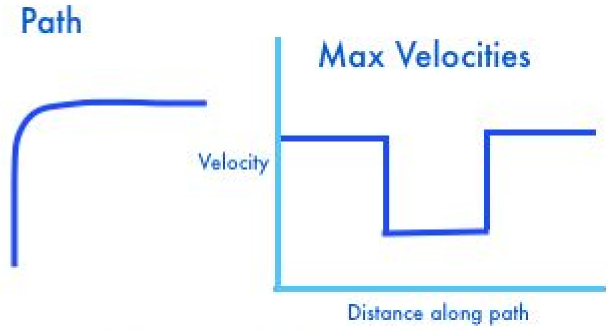
Curvatures and Velocities

- Story time.
- Slow down around turn
- Determine the curvature (rate of turn)
- How much to slow down
- Check the white paper for details on these steps

Velocity Profiles

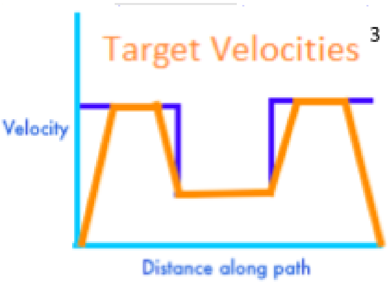


Velocity Profiles



- Slow the maximum velocity during turns to prevent tipping
- Introduce sudden acceleration

Velocity Profiles



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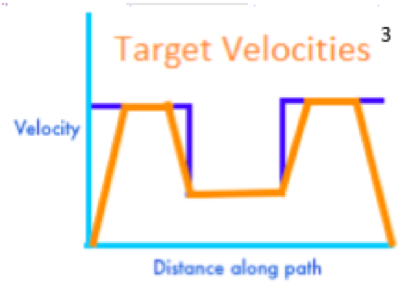
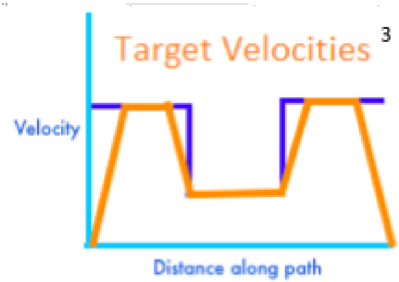


- Decelerate **before** the curve
- Re-accelerate **after** the curve
- Changing Velocity within the curve could cause tipping

Velocity Profiles



- Decelerate **before** the curve
- Re-accelerate **after** the curve
- Changing Velocity within the curve could cause tipping
- Each point has a target velocity and target acceleration

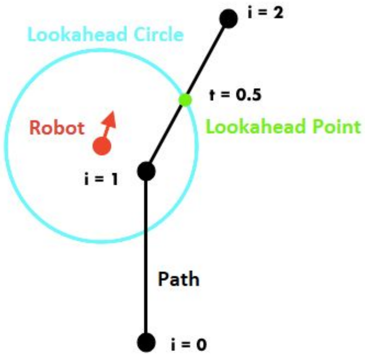
Encoding Velocity and Acceleration

- Zero (0) max velocity at the starting line

Encoding Velocity and Acceleration

- Zero (0) max velocity at the starting line
- Rate limit acceleration during runtime instead of path generation
- Use max velocity of next point instead of current point

Following the Path



Following The Path

- Know the current location using odometry
- Find the closest way point along the path
- Find the lookahead point
- Drive in an arc from current location to lookahead point
- Calculate the target left and right wheel velocities
- Use a control loop to achieve the target left and right wheel velocities

Closest Point

- cur the current location of the robot
- min the point with minimal distance from cur
- prev the previous point of minimal distance
- distance the Cartesian distance formula:
  $distance(A, B) \rightarrow \{\sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}\}$

```
min := prev;
for point in path from prev to end:
    if(distance(min, cur) > distance(point, cur)):
        min := point;
```

Lookahead Point

- Robot attempts to drive towards this point
- Follows the path as the point keeps moving forward

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- Robot attempts to drive towards this point
- Follows the path as the point keeps moving forward
- `lookahead distance` is the distance in front of the robot where the lookahead point is calculated
- Intersection of a "lookahead" circle with the path.
- two intersection points, choose the one farther in front of the robot

Lookahead Point

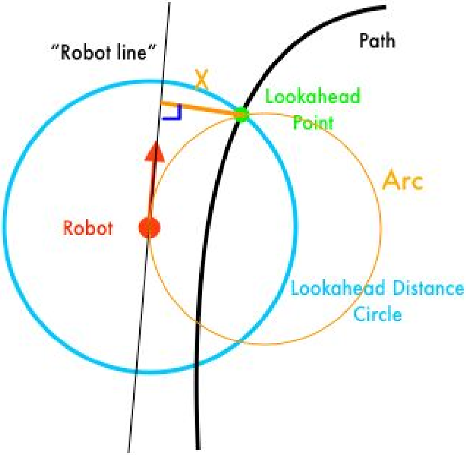
```
loc := the Robot's current location (odometry);
d := the Lookahed Distance;
n := the nearest point in the path;
segments := the lines between points in the path;
intersections := [ empty list ];
for each segment in segments from n to end:
    a, b := intersection(segment, loc, d);
    if(a != null): intersections.append(a);
    if(b != null): intersections.append(b);
lookahead_point := segments.last();
```

Check the white paper for details on the intersection of a circle and line.

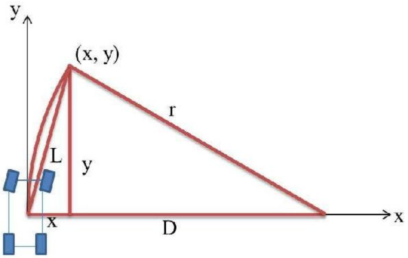
Choosing a Lookahead Distance

- Shorter lookahead for curvy paths
- Longer lookahead for smoothing a bit
- Dawgma used a distance of 12 to 25 inches
- Consider varying the lookahead distance within the path

Arc Towards the Lookahead Point



Curvature of the Arc



- Robot at the origin traveling along the Y-axis
- (X, Y) is the lookahead point
- L is the direct path to the lookahead point
- but we want to drive the arc around L
- r is the radius of the arc

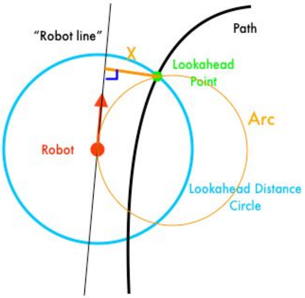
Stemming from the Pythagorean Equation

$$L = \sqrt{X^2 + Y^2} \tag{4}$$

$$r = L^2/(2X) \tag{5}$$

Curvature (C) is $1/r$

On the Field



- P is the lookahead point
- R is the robot location (from odometry)
- A is the robots angle

The robot is traveling along the line:

$$0 = -tan(A)x + y + tan(A)R_x - R_y \quad (6)$$

We can calculate X as:

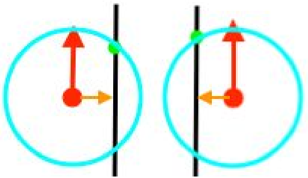
$$X = \frac{|-tan(A)P_x + P_y + tan(A)R_x - R_y|}{\sqrt{-tan(A)^2 + 1}} \quad (7)$$

and Y as:

$$Y = \sqrt{\sqrt{(P_x - R_x)^2 + (P_y - R_y)^2} - X^2} \quad (8)$$

From here we can use curvature as calculated earlier.

Which direction to turn



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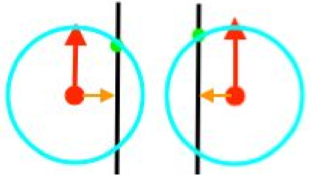


The direction to turn can be taken as the sign of the vector cross product:

$$Red \times Orange \tag{9}$$

Left if negative, right if positive.

Robot Velocity

- As fast as the robot can go

Robot Velocity

- As fast as the robot can go
- Without falling over.

Robot Velocity

- As fast as the robot can go
- Without falling over.
- Early in the season, we traveled the entire path at the maximum velocity of the sharpest turn.
- Later in the season, we added the ability to change velocities throughout the path.
- At velocity transitions we would calculate an acceleration or deceleration.

Wheel Velocities

Have:

- Target Velocity (V) of the robot
- Target curvature (ω) of the robot
- Track Width (T) of your robot

Want:

- Left wheel velocity (L)
- Right wheel velocity (R)

Wheel Velocities

Mathematical Model of a Tank Drive:

$$V = (L + R)/2 \tag{10}$$

$$\omega = (L - R)/T \tag{11}$$

$$V = \omega/C \tag{12}$$

Now we isolate L and R

$$L = V\frac{2 + CT}{2} \tag{13}$$

$$R = V\frac{2 - CT}{2} \tag{14}$$

Controlling the Wheels

- Combined Feed Forward and Feed Backward Controller
- Individually control left and right wheel speed based on Rotary Encoder velocities.
- PWM output
- Desired Velocity and Desired Acceleration

Feed Forward

- K_v proportional constant for target velocity (V)
- K_a proportional constant for target acceleration (A)

$$FF = K_v * V + K_a * A \tag{15}$$

Feed Backward

- Corrects error between actual velocity (M) and target velocity.
- K_p is the feed backwards proportional constant.

$$FB = K_p * (V - M) \tag{16}$$

Combined to get PWM output (O):

$$O = FF + FB = K_v * V + K_a * A + K_p * (V - M) \tag{17}$$

Choosing Proportional Constants

- start with a straight line path
- Set K_v approximately equal to $1/V_{max}$
- Set K_a and K_p to zero (0)
- Adjust K_v until a target velocity and the actual velocity match.

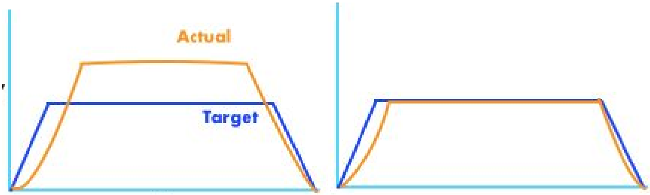


Figure: graphs of velocity vs. time. Left is at the start. right is with K_v tuned

Choosing Proportional Constants

- Set K_a to 0.002
- Adjust K_a until the acceleration lines on the graph are roughly straight

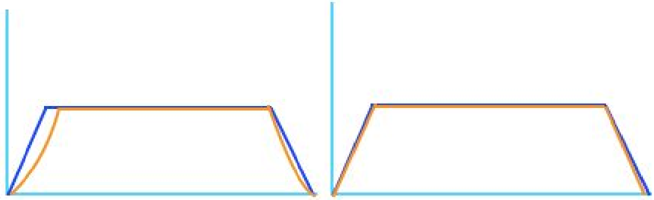


Figure: graphs of velocity vs. time. Left is at the start. right is with K_v tuned

Choosing Proportional Constants

- Set K_p to 0.01
- Adjust K_a as needed until the actual line covers the desired line
- Too much feed backwards will cause “jitteryness”

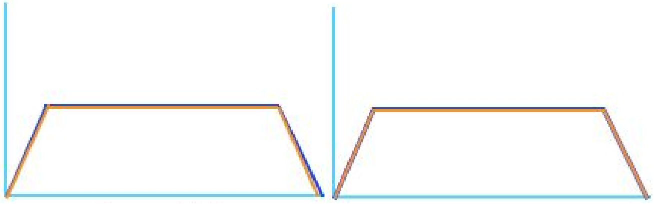
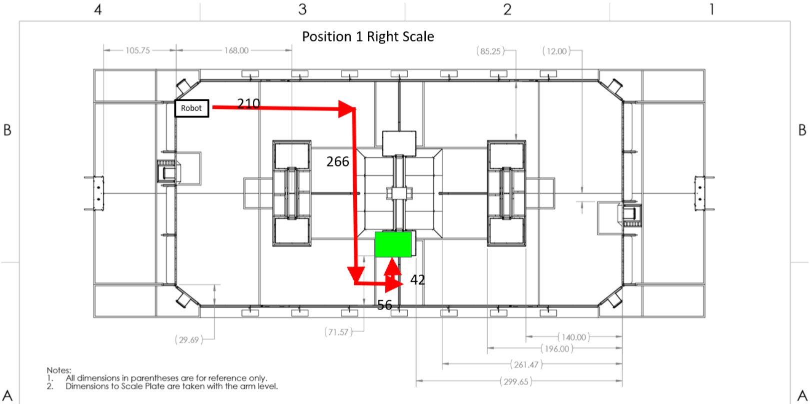


Figure: graphs of velocity vs. time. Left is at the start. right is with K_v tuned

Visualizing a Path

- PowerPoint with arrows
- Path Drawing Tool
- Path Simulation Tool

Visualizing Paths in PowerPoint



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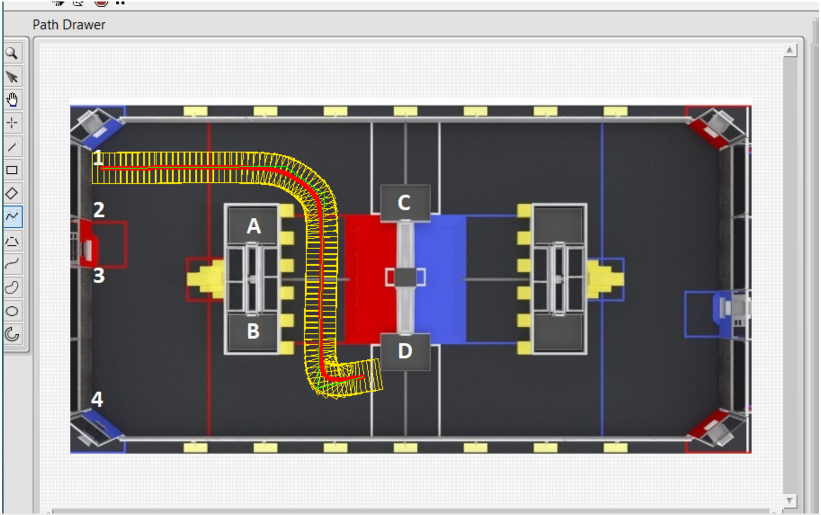

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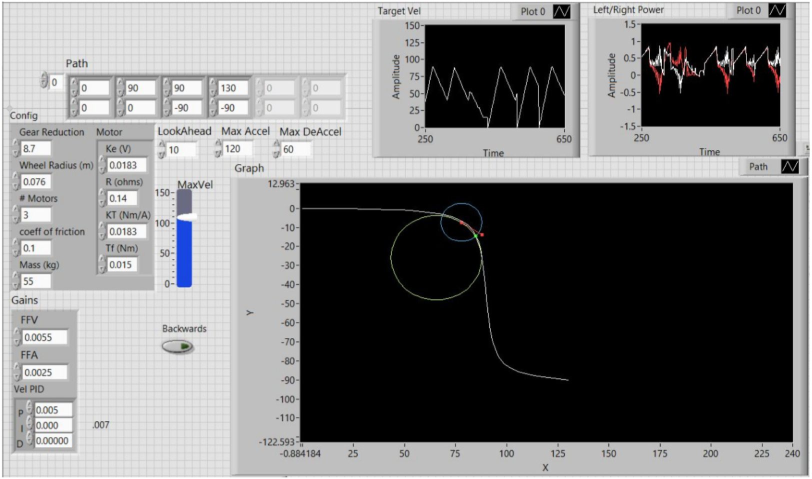

Contact and Links

- The 1712 Pure Pursuit White Paper: https://www.chiefdelphi.com/media/papers/3488
- Dawgma's 2018 Code Repository: https://github.com/Dawgma-1712/new-FRC-2018
- Chief Delphi Discussion: https://www.chiefdelphi.com/forums/showthread.php?t=166214
- Dawgma Email: frc1712@gmail.com
- My Email: kimee.i.model@gmail.com