District Points Analysis: Mathematical Locks for Advancement

Brandon Liatys, Antonio Papa

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1 Abstract

This paper describes an algorithm for computing if, partway through the season, a given team is mathematically certain of reaching their District Championship, regardless of how many points they or other teams earn at future events.

It works by finding the least amount of points required to kick a team below the cutoff rank needed to reach District Championship, and seeing if it is possible for that many points to be earned in the remaining district events. Each district event can only award a certain number of points, so if there are not enough points available to kick a certain team below the cutoff rank, then that team is mathematically guaranteed to qualify for the district championship.

2 Algorithm

2.1 Overview

The heart of the algorithm is a simple statement containing two values:

Minimum Points needed to Eliminate Team > Total Points Remaining in the District

If this statement returns true, then the team in question is considered a mathematical lock. Calculating the two values is described in the next two sections. Significant effort was put into looking for ways to logically increase the amount of teams that are locked at any point of time, while still being conservative to prevent a "false lock". For example, if we can find ways to decrease the Total Points Remaining, then more teams will be considered a lock. We looked at ways we could reduce this number by making a few assumptions that we believe are robust. Any assumptions that we made will be detailed in their sections below.

2.2 Calculating the Minimum Points to Eliminate a Team

Teams may qualify for the District Championship either by points, or by winning the Impact Award. Since each District Event has one Impact Award winner, we can subtract the number of total spots available at the District Championship by the number of District Events to find the number of points spots available.

Let's call this number N, so the top N teams in a district qualify for the District Championship by points. We need exclude current Impact Award winners, and then examine each team currently in the top N and calculate how other teams could pass them so that they will no longer be in the top N, and find the minimum number of points that those other teams need to earn to make that happen.

Let's take team A, that is ranked R_A, where $R_A \leqslant N$, meaning that this is a team ranked above the cutoff. The number of teams that need to pass them so their new rank is below N would be $N + 1 - R_A$. For example, if they are ranked 21st, and 60 teams qualify for

the District Championship using points, then 40 teams would need to pass them for them to rank 61st and not qualify.

$$N + 1 - R_A = \text{Teams to Pass}$$

$$60 + 1 - 21 = 40$$

Then we find the minimum total number of points needed to get all of those other teams to pass the team in question. We create the worst-case assumption that all tiebreakers go against team A, so each team that needs to pass team A only needs to match them in the number of points for them to get eliminated.

The points needed for a team B to pass team A is simply team A's points (P_A) minus team B's points (P_B). For example, if the 21st ranked team has 90 district points, and the 60th ranked team has 40 district points, then the 60th ranked team needs 50 more points to pass the 21st ranked team (90 - 40).

$$P_A - P_B = \text{Points to Pass}$$

$$90 - 40 = 50$$

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This is the amount of points needed for B to pass A in the rankings, but we need $N + 1 - R_A$ teams to pass team A for team A to be eliminated. To find this we repeat the calculation for the next highest ranked $N + 1 - R_A$ teams that still have a district event scheduled. We skip over any teams that already played their two events that count for points, since those teams can't earn more points. We also skip over any teams that have already won the Impact Award, since they will qualify regardless of how many points they earn, and do not take up a points spot.

$$\sum_{\text{Team } X}^{N+1-R_A \text{ Teams}} (P_A - P_X) = \text{Minimum Points to Eliminate Team } A$$

Here's an example calculation. Say that the top 60 teams qualify for the District Championship using points, and the current rankings are as follows:

Rank	Team	Points	Remaining Events
58	A	70 points	0
59	B	68 points	1
60	C	64 points	0
61	D	62 points	1
62	E	61 points	1

Table 1: Example Rankings

Team A is ranked 58th, so three teams need to pass them for them to be eliminated. The next three teams with a remaining event are B, D and E. Team C already played both of their events, so they can't pass team A, and aren't considered.

Team B could pass A if they earn 2 more points, D could pass them with 8 more points, and E could pass them with 9 more points. Therefore it would take a minimum of 19 points for A to be eliminated, and not qualify for the District Championship. This is the first value needed for our calculation.

$$(P_A - P_B) + (P_A - P_D) + (P_A - P_E) = \text{Minimum Points to Eliminate Team A}$$

$$(70 - 68) + (70 - 62) + (70 - 61) = 19$$

2.3 Calculating the Total Points Remaining

To find the total points remaining in the district we need to add up all of the points that could come from qualification matches, alliance selection, playoffs and awards for each individual event.

Alliance Selection Alliance captains earn points equal to 17 minus the alliance captain number (e.x. 14 points for alliance 3 captain). Summing all points for the captains together, there are 100 points for alliance captains.

Teams that are drafted earn points in a similar way, equal to 17 minus the draft order acceptance number. Instead of only 8 teams like alliance captains, 16 teams are drafted. Summed together, there are 136 points for alliance draft acceptance.

This results in $100 + 136 = 236$ points available for alliance selections at each event.

Playoffs Playoff points are given out based on which place an alliance finishes at.

Place	Points Per Team	Points Per Alliance
Winners	30 points	90 points
Finalists	20 points	60 points
Third Place	13 points	39 points
Fourth Place	7 points	21 points

Table 2: Playoff points

This gives us $90 + 60 + 39 + 21 = 210$ total playoff points available at each event.

However it is possible for extra points to be created when backup robots are used because of rounding. Robots that play in only some of their alliance's wins have points earned in proportion to the number of wins they played in, rounded up. This rounding means that each alliance except for the winners can create an extra playoff point.

To see why each alliance can create an extra point from rounding, consider one robot that was subbed out for some of it's alliance's wins. The substitute robot would get some fraction of the points, and the original robot would get the remaining fraction. If one robot had it's points rounded up, and the other rounded down, then we would have no new points created. However, because the points for both robots are rounded upwards, a point could be created. For example, 2.333 and 4.667 would be rounded up to 3 and 5, for a total of 8 points, instead of the original 7 points.

This rounding only applies for the double-elimination rounds, so we can ignore the finals. The upper bracket winner gets 20 points, and has three wins, so any backup there must create an extra point from rounding. The lower bracket winner gets 20 points, but always has four wins, so a backup robot cannot create an extra point from rounding as 20 is divisible by four. The next highest ranking alliances get 13 and 7 points each. Those two will create an extra rounding point if they use a backup robot since 13 and 7 are prime numbers. So in total three points can be created from the use of backup robots and points rounding.

Including the three possible points generated from rounding, we have 213 total playoff points available at each event.

$$\text{Total Playoff Points} = 90 + 60 + 39 + 21 + 3 = 213$$

Awards Most awards are worth 5 points each, with Engineering Inspiration and Rookie All Star worth 8 points, and Impact worth 10 points.

The 12 awards worth 5 points are Autonomous, Creativity, Excellence in Engineering, Gracious Professionalism, Imagery, Industrial Design, Innovation in Control, Judges', Quality, Team Spirit, Sustainability, and Rookie Inspiration.

Adding up the 12 awards worth 5 points, the two worth 8, and the Impact award worth 10 gives a total of 86 points from awards.

However we can make some adjustments to lower that number. Winners of the Impact Award qualify directly to the District Championship, so the points that they earn don't actually affect the number of teams qualifying on points. If a region has 68 spots at the District Championship and 8 District Events, then the District Championship has 60 points spots (N) and 8 Impact Award spots. We can ignore the points earned from the Impact Award here, because that team will qualify anyway. When making this adjustment it is important to remember to calculate the "Minimum Points To Eliminate Team A" by excluding the Impact Award winners from the rankings and the number of teams that qualify to the District Championship.

Additionally, two awards can only be won by rookie teams - Rookie All Star and Rookie Inspiration. So if there are no rookie teams at an event we can remove the points from these two awards. If there is only one rookie team at an event we can remove the 5 points for the Rookie Inspiration award, since a team can't win both awards at a single event.

Those adjustments give us a sum of total award points for each event as shown in the following table:

Number of Rookie Teams	Total Award Points
2 or more rookies	76
1 rookie	71
0 rookies	63

Table 3: Award points

Qualifications Qualification points follow a complex formula defined in the game manual based on how a team ranked at an event and the number of teams at the event. To calculate the total we apply the formula for every rank at the event and sum of the results. For our purposes, we will assume the formula is $f(r, m)$, where r signifies a team's rank and m is the total number of teams attending the event.

$$\sum_{r=1}^{m} f(r, m) = \text{Total Qualification Points}$$

We also can make an additional adjustment to shrink the number of points remaining. It is impossible for a team to get zero points, even the last placed team is guaranteed to get a certain number of points. At most events this is 4 points, but it could be 5 at very small events or 3 at very large ones. This means that we can add that number of points to every team's point total before the event even begins and remove them from the number of qualification points remaining (because they are guaranteed to earn that number of points). This gives us a new formula:

$$\sum_{r=1}^{m} (f(r, m) - f(m, m)) = \text{Total Qualification Points}$$

When using this approach it's important to add these points to each team's existing earned points and re-rank the teams before doing the "Minimum Points to Eliminate Team" calculation.

2.4 Final Result

Now we can sum up the Total Qualification Points, Selection Points, Playoff Points and Award points to find the Total Points Remaining.

If the Total Points Remaining is less than team A's Minimum Points to Eliminate, then team A is guaranteed to qualify for the District Championship. There is no way to distribute the remaining points in a manner that would push team A below the points cutoff. The above process is repeated for every team in ranks 1 to N, where N is the number of points spots at the District Championship, as described in section 2.2.

3 Conclusions

This approach has been calculated first in Excel workbooks posted on ChiefDelphi threads, and then in an automated way on `frclocks.com` for ten seasons. It is a conservative approach that identifies teams that are certain to qualify for the District Championship. It is not a probabilistic method.

The `frclocks.com` website also runs a lock calculation during the course of events, removing Qualification, Selection and Playoff points as events are in progress. That methodology is not covered by this whitepaper, but it expands on the methods described above. The website also does a similar live display to calculate who has qualified for the World Championship, and the adjustments to the algorithm made for that use case are also not covered in this paper.

4 Acknowledgments

This algorithm is built on work by Brian Lucas in his 2013 paper "2013 MAR Champ Clinch Berth", `https://www.chiefdelphi.com/media/papers/2805`