Robot Tipping Point Accelerations

An approximated relationship between drivetrain acceleration and elevator acceleration

Team Makers Assemble #5951

Background:

While designing an angled elevator for the 2023 offseason, prior observation throughout the season cautioned us of tipping over. Therefore, we wanted to calculate at what accelerations can our upcoming robot operate at without tipping.

Goal:

Find the tipping points of a robot through a relationship between robot drivetrain acceleration and elevator acceleration.

Defining Variables:

$N_1 - Front\ Wheel\ Normal$

$N_2 - Back\ Wheel\ Normal$

$F_{CM} - Robot\ Body\ Forces$

$F_{elevator} - Elevator\ opening \backslash closing\ forces$

$m_{tot} - total\ robot\ mass$

$m - elevator\ mass$

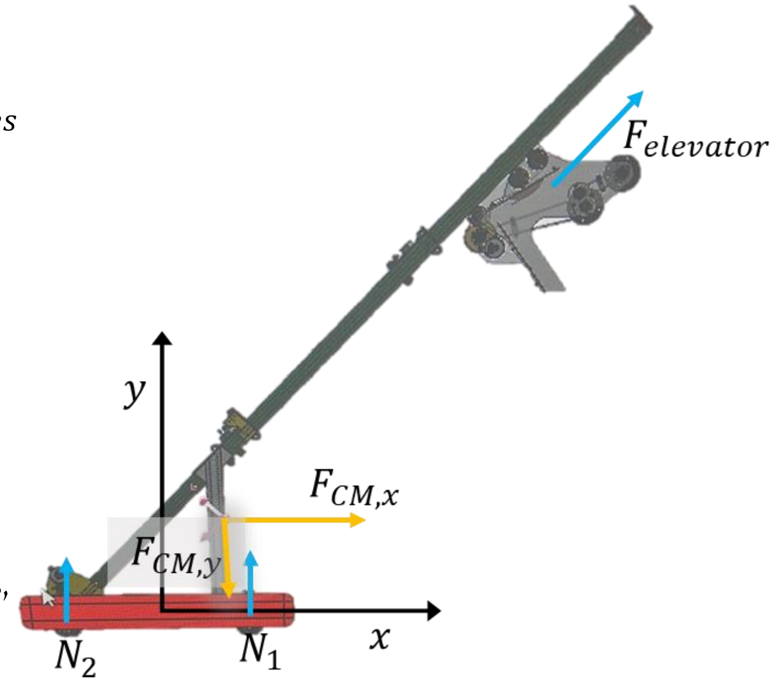
$\theta - elevator\ angle$

$g - gravitational\ constant$

$x_{CM}, y_{CM} - Center\ of\ Mass\ coordinates$

$x_{elevator}, y_{elevator} - elevator\ coordinates$

Figure 1 - Robot Free Body Diagram



Assumptions:

1. Due to the position of our center of mass, the tipping concern is tipping forwards. (Clockwise relative to figure 1)
2. At the critical tipping point, the back -wheel would stop touching the floor, meaning $N_2 = 0$.
3. The condition to be met to tip forwards is when $\Sigma M \leq 0$ *
4. To simplify the problem – we will only look at the worse case position of the center of mass (when the elevator is fully open). Assuming the center of mass to be constant.

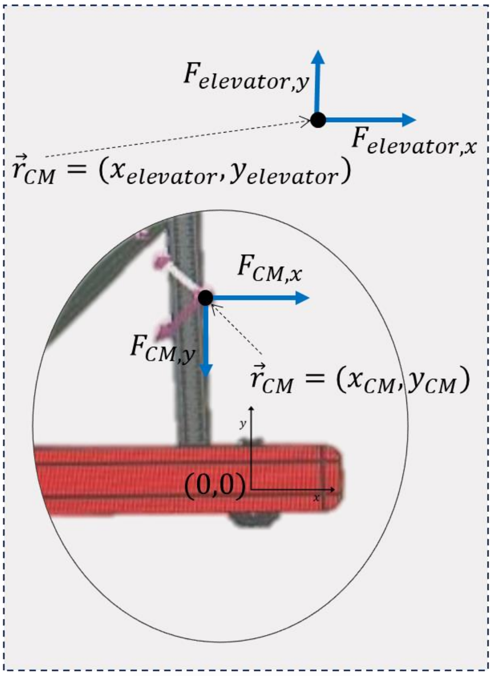
* leq (<) and not geq (>) torque value due to how we defined our axis

Equation Development:

$$\vec{F}_{elevator} = m \cdot \vec{a}_{elevator}$$
$$F_{CM,y} = -m_{tot} \cdot g$$
$$F_{CM,x} = -m_{tot} \cdot a_{drivetrain}$$

$F_{CM,x}$ is a fictitious force due to the inertial frame of the system, the acceleration being in similar magnitude but opposite direction of the drivetrain acceleration.

Next, we will calculate the total torque about the robot pivot point while tipping (the front wheel center)



$$\Sigma M = \Sigma(F \times r) = \left(F_{CM,x} \cdot y_{CM} - F_{CM,y} \cdot x_{CM}\right) + \left(F_{elevator,x} \cdot y_{elevator} - F_{elevator,y} \cdot x_{elevator}\right) = 0$$

$$(-m_{tot} \cdot a_{robot} \cdot y_{CM} - (-m_{tot} \cdot g \cdot x_{CM})) + \left(F_{el\ldots,x} \cdot y_{el\ldots} - F_{el\ldots,y} \cdot x_{el\ldots}\right) = 0$$

We can simplify the equation and rewrite it in the form of:

$$A \cdot a_{robot} + B \cdot a_{elevator} + C = 0$$

Such that:

$$A = -m_{tot} \cdot y_{CM}$$

$$B = m_{elevator} \cdot (cos(\theta) \cdot y_{elevator} - sin(\theta) \cdot x_{elevator})$$

$$C = m_{tot} \cdot g \cdot x_{CM}$$

Therefore, if we want the robot <u>not</u> to tip over the following condition must be met:

$$A \cdot a_{robot} + B \cdot a_{elevator} + C > 0$$

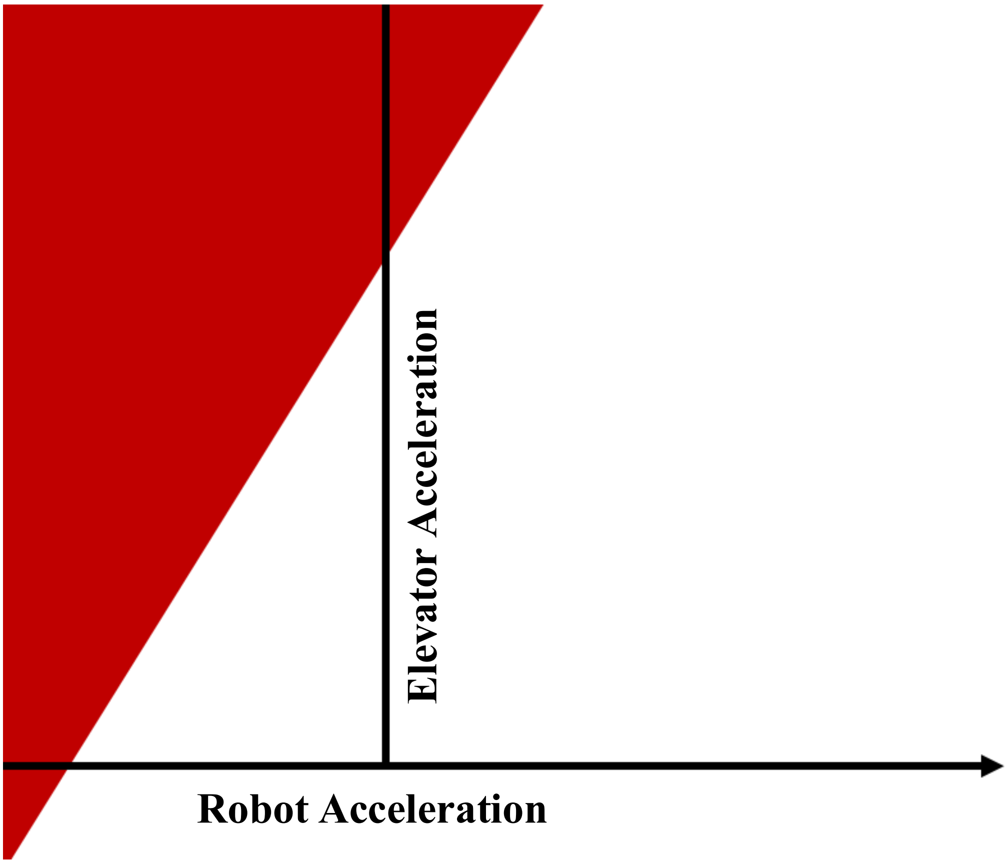
Meaning:

$$\boldsymbol{a_{elevator} > -\frac{A \cdot a_{robot} + C}{B}}$$

And this is the allowable elevator acceleration given the drivetrain acceleration of the robot!

(Due to our 4th assumption, this is most accurate for a fully open elevator)

Plotting the equation should grant a graph that looks the following:



The red zone is the flipping zone, white zone is safe. One can see that as the robot is accelerating backwards, the limit on the elevator acceleration gets tighter. A critical point on the graph is the x-axis intersection – where the robot would tip over with a static elevator.

In our offseason using the following evaluation has led us to increase the robot weight (in the right places - to shift our center of mass) and assisted us in coding proper acceleration limits for the drivetrain when the elevator is open.

We hope this short white paper will help you. If you have any questions or if you found a mistake, feel free to contact us through Daniel.oreg@gmail.com, or @oregano on chief delphi (: